

# Mathematica 11.3 Integration Test Results

on the problems in the test-suite directory "5 Inverse trig functions\5.5 Inverse secant"

Test results for the 174 problems in "5.5.1 u (a+b arcsec(c x))^n.m"

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b \operatorname{ArcSec}[c x])^3 dx$$

Optimal (type 4, 236 leaves, 11 steps):

$$\frac{b^2 x (a + b \operatorname{ArcSec}[c x])}{c^2} - \frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x^2 (a + b \operatorname{ArcSec}[c x])^2}{2 c} + \frac{1}{3} x^3 (a + b \operatorname{ArcSec}[c x])^3 +$$

$$\frac{i b (a + b \operatorname{ArcSec}[c x])^2 \operatorname{ArcTan}[e^{i \operatorname{ArcSec}[c x]}]}{c^3} - \frac{b^3 \operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{c^2 x^2}}\right]}{c^3} - \frac{i b^2 (a + b \operatorname{ArcSec}[c x]) \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSec}[c x]}]}{c^3} +$$

$$\frac{i b^2 (a + b \operatorname{ArcSec}[c x]) \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSec}[c x]}]}{c^3} + \frac{b^3 \operatorname{PolyLog}[3, -i e^{i \operatorname{ArcSec}[c x]}]}{c^3} - \frac{b^3 \operatorname{PolyLog}[3, i e^{i \operatorname{ArcSec}[c x]}]}{c^3}$$

Result (type 4, 775 leaves):

$$\begin{aligned}
& \frac{1}{6c^3} \left( 6ab^2cx - 3a^2bc^2 \sqrt{1 - \frac{1}{c^2x^2}} x^2 + 2a^3c^3x^3 + 6b^3cx \operatorname{ArcSec}[cx] - 6ab^2c^2 \sqrt{1 - \frac{1}{c^2x^2}} x^2 \operatorname{ArcSec}[cx] + 6a^2bc^3x^3 \operatorname{ArcSec}[cx] - \right. \\
& 3b^3c^2 \sqrt{1 - \frac{1}{c^2x^2}} x^2 \operatorname{ArcSec}[cx]^2 + 6ab^2c^3x^3 \operatorname{ArcSec}[cx]^2 + 2b^3c^3x^3 \operatorname{ArcSec}[cx]^3 - 6ab^2 \operatorname{ArcSec}[cx] \operatorname{Log}[1 - i e^{i \operatorname{ArcSec}[cx]}] - \\
& 3b^3 \operatorname{ArcSec}[cx]^2 \operatorname{Log}[1 - i e^{i \operatorname{ArcSec}[cx]}] + 6ab^2 \operatorname{ArcSec}[cx] \operatorname{Log}[1 + i e^{i \operatorname{ArcSec}[cx]}] + 3b^3 \operatorname{ArcSec}[cx]^2 \operatorname{Log}[1 + i e^{i \operatorname{ArcSec}[cx]}] - \\
& 3b^3 \pi \operatorname{ArcSec}[cx] \operatorname{Log}\left[\left(-\frac{1}{2} - \frac{i}{2}\right) e^{-\frac{1}{2}i \operatorname{ArcSec}[cx]} (-i + e^{i \operatorname{ArcSec}[cx]})\right] + 3b^3 \operatorname{ArcSec}[cx]^2 \operatorname{Log}\left[\left(\frac{1}{2} + \frac{i}{2}\right) e^{-\frac{1}{2}i \operatorname{ArcSec}[cx]} (-i + e^{i \operatorname{ArcSec}[cx]})\right] - \\
& 3b^3 \pi \operatorname{ArcSec}[cx] \operatorname{Log}\left[\frac{1}{2} e^{-\frac{1}{2}i \operatorname{ArcSec}[cx]} \left((1+i) + (1-i) e^{i \operatorname{ArcSec}[cx]}\right)\right] - 3b^3 \operatorname{ArcSec}[cx]^2 \operatorname{Log}\left[\frac{1}{2} e^{-\frac{1}{2}i \operatorname{ArcSec}[cx]} \left((1+i) + (1-i) e^{i \operatorname{ArcSec}[cx]}\right)\right] - \\
& 3a^2b \operatorname{Log}\left[\left(1 + \sqrt{1 - \frac{1}{c^2x^2}}\right)x\right] + 6b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right]\right] - \\
& 3b^3 \operatorname{ArcSec}[cx]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right]\right] + 3b^3 \pi \operatorname{ArcSec}[cx] \operatorname{Log}\left[-\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right]\right] - \\
& 6b^3 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right]\right] + 3b^3 \pi \operatorname{ArcSec}[cx] \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right]\right] + \\
& 3b^3 \operatorname{ArcSec}[cx]^2 \operatorname{Log}\left[\operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[cx]\right]\right] - 6ib^2(a + b \operatorname{ArcSec}[cx]) \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSec}[cx]}] + \\
& \left. 6ib^2(a + b \operatorname{ArcSec}[cx]) \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSec}[cx]}] + 6b^3 \operatorname{PolyLog}[3, -i e^{i \operatorname{ArcSec}[cx]}] - 6b^3 \operatorname{PolyLog}[3, i e^{i \operatorname{ArcSec}[cx]}]\right)
\end{aligned}$$

**Problem 63: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d + ex)^{3/2} (a + b \operatorname{ArcSec}[cx]) dx$$

Optimal (type 4, 372 leaves, 22 steps):

$$\frac{4 b e \sqrt{d+e x} (1-c^2 x^2)}{15 c^3 \sqrt{1-\frac{1}{c^2 x^2}} x} + \frac{2 (d+e x)^{5/2} (a+b \operatorname{ArcSec}[c x])}{5 e} + \frac{28 b d \sqrt{d+e x} \sqrt{1-c^2 x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{15 c^2 \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{\frac{c(d+e x)}{c d+e}}} +$$

$$\frac{4 b (2 c^2 d^2+e^2) \sqrt{\frac{c(d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{15 c^4 \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}} + \frac{4 b d^3 \sqrt{\frac{c(d+e x)}{c d+e}} \sqrt{1-c^2 x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{5 c e \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}$$

Result (type 4, 333 leaves):

$$\frac{1}{15} \left( -\frac{4 b e \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{d+e x}}{c} + \frac{6 a (d+e x)^{5/2}}{e} + \frac{6 b (d+e x)^{5/2} \operatorname{ArcSec}[c x]}{e} + \right.$$

$$\left. \left( 4 i b \sqrt{\frac{e(1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left( -7 c d (c d-e) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] + \right. \right. \right.$$

$$\left. \left. \left( 9 c^2 d^2 - 7 c d e + e^2 \right) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] - \right. \right.$$

$$\left. \left. \left. 3 c^2 d^2 \operatorname{EllipticPi}\left[1 + \frac{e}{c d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] \right) \right) \right) / \left( c^3 e \sqrt{-\frac{c}{c d+e}} \sqrt{1-\frac{1}{c^2 x^2}} x \right)$$

**Problem 64: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{d+e x} (a+b \operatorname{ArcSec}[c x]) dx$$

Optimal (type 4, 315 leaves, 15 steps):

$$\frac{2 (d + e x)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 e} + \frac{4 b \sqrt{d + e x} \sqrt{1 - c^2 x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 - c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]}{3 c^2 \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{\frac{c (d + e x)}{c d + e}}} +$$

$$\frac{4 b d \sqrt{\frac{c (d + e x)}{c d + e}} \sqrt{1 - c^2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1 - c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]}{3 c^2 \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + e x}} + \frac{4 b d^2 \sqrt{\frac{c (d + e x)}{c d + e}} \sqrt{1 - c^2 x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1 - c x}}{\sqrt{2}}\right], \frac{2 e}{c d + e}\right]}{3 c e \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{d + e x}}$$

Result (type 4, 277 leaves):

$$\frac{1}{3 e} 2 \left( a (d + e x)^{3/2} + b (d + e x)^{3/2} \operatorname{ArcSec}[c x] + \frac{1}{c^2 \sqrt{-\frac{c}{c d + e}} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right.$$

$$2 i b \sqrt{\frac{e (1 + c x)}{-c d + e}} \sqrt{\frac{e - c e x}{c d + e}} \left( (-c d + e) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d + e}} \sqrt{d + e x}\right], \frac{c d + e}{c d - e}\right] + \right.$$

$$\left. \left. (2 c d - e) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d + e}} \sqrt{d + e x}\right], \frac{c d + e}{c d - e}\right] - c d \operatorname{EllipticPi}\left[1 + \frac{e}{c d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d + e}} \sqrt{d + e x}\right], \frac{c d + e}{c d - e}\right]\right) \right)$$

**Problem 65: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSec}[c x]}{\sqrt{d + e x}} dx$$

Optimal (type 4, 212 leaves, 9 steps):

$$\frac{2\sqrt{d+ex} (a+b \operatorname{ArcSec}[cx])}{e} + \frac{4b \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1-c^2x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{c^2 \sqrt{1-\frac{1}{c^2x^2}} x \sqrt{d+ex}} +$$

$$\frac{4bd \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1-c^2x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{ce \sqrt{1-\frac{1}{c^2x^2}} x \sqrt{d+ex}}$$

Result (type 4, 212 leaves):

$$\frac{1}{2} \left( a \sqrt{d+ex} + b \sqrt{d+ex} \operatorname{ArcSec}[cx] + \frac{1}{c \sqrt{-\frac{c}{cd+e}} \sqrt{1-\frac{1}{c^2x^2}} x} {}_2F_1\left[\begin{matrix} 1 \\ 1 \end{matrix}; \begin{matrix} -\frac{c}{cd+e} \end{matrix}; -\frac{e(1+cx)}{-cd+e} \right] \sqrt{\frac{e-cex}{cd+e}} \right. \\ \left. \left( \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right], \frac{cd+e}{cd-e}\right] - \operatorname{EllipticPi}\left[1 + \frac{e}{cd}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{cd+e}} \sqrt{d+ex}\right], \frac{cd+e}{cd-e}\right] \right) \right)$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \operatorname{ArcSec}[cx]}{(d+ex)^{5/2}} dx$$

Optimal (type 4, 298 leaves, 12 steps):

$$-\frac{4be(1-c^2x^2)}{3cd(c^2d^2-e^2) \sqrt{1-\frac{1}{c^2x^2}} x \sqrt{d+ex}} - \frac{2(a+b \operatorname{ArcSec}[cx])}{3e(d+ex)^{3/2}} + \\ \frac{4b\sqrt{d+ex} \sqrt{1-c^2x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{3d(c^2d^2-e^2) \sqrt{1-\frac{1}{c^2x^2}} x \sqrt{\frac{c(d+ex)}{cd+e}}} - \frac{4b \sqrt{\frac{c(d+ex)}{cd+e}} \sqrt{1-c^2x^2} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{3cde \sqrt{1-\frac{1}{c^2x^2}} x \sqrt{d+ex}}$$

Result (type 4, 326 leaves):

$$\frac{1}{3e} \left[ -\frac{a}{(d+ex)^{3/2}} + \frac{2bc e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x}{(c^2 d^3 - de^2) \sqrt{d+ex}} - \frac{b \operatorname{ArcSec}[cx]}{(d+ex)^{3/2}} - \right. \\ \left. \left( 2i b \sqrt{\frac{e(1+cx)}{-cd+e}} \sqrt{\frac{e-cex}{cd+e}} \left( -cd \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right], \frac{cd+e}{cd-e} \right] + cd \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right], \frac{cd+e}{cd-e} \right], \right. \right. \right. \\ \left. \left. \left. \frac{cd+e}{cd-e} \right) + (cd+e) \operatorname{EllipticPi}\left[ 1 + \frac{e}{cd}, i \operatorname{ArcSinh}\left[ \sqrt{-\frac{c}{cd+e}} \sqrt{d+ex} \right], \frac{cd+e}{cd-e} \right] \right) \right] / \left( d^2 \left( -\frac{c}{cd+e} \right)^{3/2} (cd+e)^2 \sqrt{1 - \frac{1}{c^2 x^2}} x \right)$$

**Problem 68: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSec}[cx]}{(d+ex)^{7/2}} dx$$

Optimal (type 4, 540 leaves, 19 steps):

$$-\frac{4be(1-c^2x^2)}{15cd(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x(d+ex)^{3/2}} - \frac{16bce(1-c^2x^2)}{15(c^2d^2-e^2)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4be(1-c^2x^2)}{5cd^2(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \\ \frac{2(a+b\operatorname{ArcSec}[cx])}{5e(d+ex)^{5/2}} + \frac{4b(7c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1-c^2x^2}\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{15(c^2d^3-de^2)^2\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{\frac{c(d+ex)}{cd+e}}} - \\ \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{15d(c^2d^2-e^2)\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{4b\sqrt{\frac{c(d+ex)}{cd+e}}\sqrt{1-c^2x^2}\operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right]}{5cd^2e\sqrt{1-\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

Result (type 4, 407 leaves):

$$\frac{1}{15 e} 2 \left( -\frac{3 a}{(d+e x)^{5/2}} + \frac{2 b c e^2 \sqrt{1 - \frac{1}{c^2 x^2}} x (-e^2 (4 d + 3 e x) + c^2 d^2 (8 d + 7 e x))}{(c^2 d^3 - d e^2)^2 (d+e x)^{3/2}} - \frac{3 b \text{ArcSec}[c x]}{(d+e x)^{5/2}} + \right. \\ \left. \left( 2 i b \sqrt{\frac{e(1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \left( c d (7 c^2 d^2 - 3 e^2) \text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] - \right. \right. \right. \\ \left. \left. \left. c d (6 c^2 d^2 - c d e - 3 e^2) \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] - 3 (c d-e) (c d+e)^2 \right. \right. \right. \\ \left. \left. \left. \text{EllipticPi}\left[1 + \frac{e}{c d}, \text{i ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right]\right) \right) \right) / \left( d^3 (c d-e) \left(-\frac{c}{c d+e}\right)^{3/2} (c d+e)^3 \sqrt{1 - \frac{1}{c^2 x^2}} x \right)$$

**Problem 96: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^5 (a + b \text{ArcSec}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 608 leaves, 31 steps):

$$\frac{b \sqrt{1 - \frac{1}{c^2 x^2}} x}{2 c e^2} + \frac{d (a + b \text{ArcSec}[c x])}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 (a + b \text{ArcSec}[c x])}{2 e^2} + \frac{b d \text{ArcTan}\left[\frac{\sqrt{c^2 d+e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}}\right]}{2 e^{5/2} \sqrt{c^2 d+e}} - \frac{d (a + b \text{ArcSec}[c x]) \text{Log}\left[1 - \frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d+e}}\right]}{e^3} - \\ \frac{d (a + b \text{ArcSec}[c x]) \text{Log}\left[1 + \frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d+e}}\right]}{e^3} - \frac{d (a + b \text{ArcSec}[c x]) \text{Log}\left[1 - \frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d+e}}\right]}{e^3} - \frac{d (a + b \text{ArcSec}[c x]) \text{Log}\left[1 + \frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d+e}}\right]}{e^3} + \\ \frac{2 d (a + b \text{ArcSec}[c x]) \text{Log}\left[1 + e^{2 \text{i ArcSec}[c x]}\right]}{e^3} + \frac{\text{i} b d \text{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d+e}}\right]}{e^3} + \frac{\text{i} b d \text{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d+e}}\right]}{e^3} + \\ \frac{\text{i} b d \text{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d+e}}\right]}{e^3} + \frac{\text{i} b d \text{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\text{i ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d+e}}\right]}{e^3} - \frac{\text{i} b d \text{PolyLog}\left[2, -e^{2 \text{i ArcSec}[c x]}\right]}{e^3}$$

Result (type 4, 1362 leaves):

$$\frac{a x^2}{2 e^2} - \frac{a d^2}{2 e^3 (d + e x^2)} - \frac{a d \operatorname{Log}[d + e x^2]}{e^3} + b \left( \frac{x \left( -\sqrt{1 - \frac{1}{c^2 x^2}} + c x \operatorname{ArcSec}[c x] \right)}{2 c e^2} \right) +$$

$$\frac{i d^{3/2} \left( -\frac{\operatorname{ArcSec}[c x]}{i \sqrt{d} \sqrt{e + e x}} + \frac{i \left( \frac{\operatorname{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}} - \frac{\operatorname{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left( \sqrt{e + c \left( i c \sqrt{d} - \sqrt{-c^2 d - e} \sqrt{1 - \frac{1}{c^2 x^2}} \right) x\right]}{\sqrt{-c^2 d - e} \left( \sqrt{d} - i \sqrt{e} x \right)} \right)}{\sqrt{-c^2 d - e}} \right)}{\sqrt{d}} \right)}{4 e^{5/2}} - \frac{i d^{3/2} \left( -\frac{\operatorname{ArcSec}[c x]}{-i \sqrt{d} \sqrt{e + e x}} - \frac{i \left( \frac{\operatorname{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}} - \frac{\operatorname{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left( -\sqrt{e + c \left( i c \sqrt{d} + \sqrt{-c^2 d - e} \sqrt{1 - \frac{1}{c^2 x^2}} \right) x\right]}{\sqrt{-c^2 d - e} \left( \sqrt{d} + i \sqrt{e} x \right)} \right)}{\sqrt{-c^2 d - e}} \right)}{\sqrt{d}} \right)}{4 e^{5/2}} -$$

$$\frac{1}{2 e^3} i d \left( 8 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[c x]\right]}{\sqrt{c^2 d + e}}\right] - 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i \left( \sqrt{e} - \sqrt{c^2 d + e} \right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] \right) -$$



$$\begin{aligned}
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]-2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]+ \\
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]+2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1+e^{2 i \operatorname{ArcSec}[c x]}\right]- \\
& 2 \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]-2 \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right]- \\
& \frac{1}{2 e^3} i d \left( 8 \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[c x]\right]}{\sqrt{c^2 d+e}}\right]-2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]- \right. \\
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]-2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]+ \\
& \left. 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]+2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1+e^{2 i \operatorname{ArcSec}[c x]}\right]- \right)
\end{aligned}$$

$$2 \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right]$$

**Problem 97: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^3 (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 570 leaves, 29 steps):

$$\begin{aligned}
& - \frac{a + b \operatorname{ArcSec}[c x]}{2 e \left( e + \frac{d}{x^2} \right)} - \frac{b \operatorname{ArcTan}\left[ \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x} \right]}{2 e^{3/2} \sqrt{c^2 d + e}} + \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} + \\
& \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} - \\
& \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[ 1 + e^{2 i \operatorname{ArcSec}[c x]} \right]}{e^2} - \frac{i b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} - \frac{i b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} - \\
& \frac{i b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} - \frac{i b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} + \frac{i b \operatorname{PolyLog}\left[ 2, -e^{2 i \operatorname{ArcSec}[c x]} \right]}{2 e^2}
\end{aligned}$$

Result (type 4, 1213 leaves):

$$\frac{1}{4 e^2}$$

$$\left( \frac{2 a d}{d + e x^2} + \frac{b \sqrt{d} \operatorname{ArcSec}[c x]}{\sqrt{d} - i \sqrt{e} x} + \frac{b \sqrt{d} \operatorname{ArcSec}[c x]}{\sqrt{d} + i \sqrt{e} x} + 2 b \operatorname{ArcSin}\left[ \frac{1}{c x} \right] + 8 i b \operatorname{ArcSin}\left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTan}\left[ \frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tan}\left[ \frac{1}{2} \operatorname{ArcSec}[c x] \right]}{\sqrt{c^2 d + e}} \right] \right) +$$

$$8 i b \operatorname{ArcSin}\left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTan}\left[ \frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tan}\left[ \frac{1}{2} \operatorname{ArcSec}[c x] \right]}{\sqrt{c^2 d + e}} \right] + 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[ 1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}} \right] +$$

$$4 b \operatorname{ArcSin}\left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log}\left[ 1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}} \right] + 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[ 1 + \frac{i (-\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}} \right] +$$

$$4 b \operatorname{ArcSin}\left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log}\left[ 1 + \frac{i (-\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}} \right] + 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[ 1 - \frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}} \right] -$$

$$\begin{aligned}
& 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] + 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] - \\
& 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] - 4 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSec}[c x]}\right] - \\
& \frac{b\sqrt{e} \operatorname{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e} + c\left(i c\sqrt{d} - \sqrt{-c^2 d - e}\sqrt{1 - \frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d - e}(\sqrt{d} - i\sqrt{e}x)}\right]}{\sqrt{-c^2 d - e}} - \frac{b\sqrt{e} \operatorname{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(-\sqrt{e} + c\left(i c\sqrt{d} + \sqrt{-c^2 d - e}\sqrt{1 - \frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d - e}(\sqrt{d} + i\sqrt{e}x)}\right]}{\sqrt{-c^2 d - e}} + \\
& 2 a \operatorname{Log}[d + e x^2] - 2 i b \operatorname{PolyLog}\left[2, \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] - 2 i b \operatorname{PolyLog}\left[2, \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] - \\
& \left. 2 i b \operatorname{PolyLog}\left[2, -\frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] - 2 i b \operatorname{PolyLog}\left[2, \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec}[c x]}}{c\sqrt{d}}\right] + 2 i b \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right] \right)
\end{aligned}$$

**Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{a + b \operatorname{ArcSec}[c x]}{2 e (d + e x^2)} + \frac{b c x \operatorname{ArcTan}\left[\sqrt{-1 + c^2 x^2}\right]}{2 d e \sqrt{c^2 x^2}} - \frac{b c x \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{\sqrt{c^2 d + e}}\right]}{2 d \sqrt{e} \sqrt{c^2 d + e} \sqrt{c^2 x^2}}$$

Result (type 3, 286 leaves):

$$\frac{1}{4e} \left( \frac{2a}{d+ex^2} - \frac{2b \operatorname{ArcSec}[cx]}{d+ex^2} - \frac{2b \operatorname{ArcSin}\left[\frac{1}{cx}\right]}{d} + \frac{b\sqrt{e} \operatorname{Log}\left[\frac{4ide+4cd\sqrt{e}\left(c\sqrt{d-i}\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d+i}\sqrt{e}x\right)}\right]}{d\sqrt{-c^2d-e}} + \frac{b\sqrt{e} \operatorname{Log}\left[\frac{-4ide+4cd\sqrt{e}\left(c\sqrt{d+i}\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x}{b\sqrt{-c^2d-e}\left(\sqrt{d-i}\sqrt{e}x\right)}\right]}{d\sqrt{-c^2d-e}} \right)$$

**Problem 99: Result more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{ArcSec}[cx]}{x(d+ex^2)^2} dx$$

Optimal (type 4, 546 leaves, 24 steps):

$$\begin{aligned} & -\frac{e(a+b \operatorname{ArcSec}[cx])}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{i(a+b \operatorname{ArcSec}[cx])^2}{2bd^2} - \frac{b\sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{1-\frac{1}{c^2x^2}}}\right]}{2d^2\sqrt{c^2d+e}} - \frac{(a+b \operatorname{ArcSec}[cx]) \operatorname{Log}\left[1-\frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2d^2} \\ & \frac{(a+b \operatorname{ArcSec}[cx]) \operatorname{Log}\left[1+\frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2d^2} - \frac{(a+b \operatorname{ArcSec}[cx]) \operatorname{Log}\left[1-\frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2d^2} - \frac{(a+b \operatorname{ArcSec}[cx]) \operatorname{Log}\left[1+\frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2d^2} + \\ & \frac{i b \operatorname{PolyLog}\left[2, -\frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2d^2} + \frac{i b \operatorname{PolyLog}\left[2, \frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}-\sqrt{c^2d+e}}\right]}{2d^2} + \frac{i b \operatorname{PolyLog}\left[2, -\frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2d^2} + \frac{i b \operatorname{PolyLog}\left[2, \frac{c\sqrt{-d}e^{i \operatorname{ArcSec}[cx]}}{\sqrt{e}+\sqrt{c^2d+e}}\right]}{2d^2} \end{aligned}$$

Result (type 4, 1190 leaves):

$$\frac{1}{4d^2} \left( \frac{2ad}{d+ex^2} + \frac{b\sqrt{d} \operatorname{ArcSec}[cx]}{\sqrt{d-i}\sqrt{e}x} + \frac{b\sqrt{d} \operatorname{ArcSec}[cx]}{\sqrt{d+i}\sqrt{e}x} + 2i b \operatorname{ArcSec}[cx]^2 + \right)$$

$$\begin{aligned}
& 2 b \operatorname{ArcSin}\left[\frac{1}{c x}\right] - 8 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(-i c \sqrt{d} + \sqrt{e}\right) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[c x]\right]}{\sqrt{c^2 d + e}}\right] - \\
& 8 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(i c \sqrt{d} + \sqrt{e}\right) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[c x]\right]}{\sqrt{c^2 d + e}}\right] - 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - \\
& 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - \\
& 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \\
& 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 b \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \\
& 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + 4 a \operatorname{Log}[x] - \\
& \frac{b \sqrt{e} \operatorname{Log}\left[\frac{2 \sqrt{d} \sqrt{e}\left(\sqrt{e} + c\left(i c \sqrt{d} - \sqrt{-c^2 d - e}\right) \sqrt{1 - \frac{1}{c^2 x^2}}\right) x}{\sqrt{-c^2 d - e}\left(\sqrt{d} - i \sqrt{e} x\right)}\right]}{\sqrt{-c^2 d - e}} - \frac{b \sqrt{e} \operatorname{Log}\left[\frac{2 \sqrt{d} \sqrt{e}\left(-\sqrt{e} + c\left(i c \sqrt{d} + \sqrt{-c^2 d - e}\right) \sqrt{1 - \frac{1}{c^2 x^2}}\right) x}{\sqrt{-c^2 d - e}\left(\sqrt{d} + i \sqrt{e} x\right)}\right]}{\sqrt{-c^2 d - e}} - \\
& 2 a \operatorname{Log}[d + e x^2] + 2 i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + 2 i b \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \\
& \left. 2 i b \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + 2 i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right]\right)
\end{aligned}$$

### Problem 104: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 707 leaves, 33 steps):

$$\begin{aligned} & -\frac{b c d \sqrt{1 - \frac{1}{c^2 x^2}}}{8 e^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} - \frac{a + b \operatorname{ArcSec}[c x]}{4 e \left(e + \frac{d}{x^2}\right)^2} - \frac{a + b \operatorname{ArcSec}[c x]}{2 e^2 \left(e + \frac{d}{x^2}\right)} - \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}}\right]}{2 e^{5/2} \sqrt{c^2 d + e}} - \frac{b (c^2 d + 2 e) \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}}}\right]}{8 e^{5/2} (c^2 d + e)^{3/2}} + \\ & \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} + \\ & \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} - \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSec}[c x]}\right]}{e^3} - \frac{i b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} - \\ & \frac{i b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} - \frac{i b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} - \frac{i b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{i b \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right]}{2 e^3} \end{aligned}$$

Result (type 4, 1805 leaves):

$$-\frac{a d^2}{4 e^3 (d + e x^2)^2} + \frac{a d}{e^3 (d + e x^2)} + \frac{a \operatorname{Log}[d + e x^2]}{2 e^3} +$$

$$\begin{aligned}
 & \left( \frac{7 i \sqrt{d}}{16 e^{5/2}} \left( -\frac{\text{ArcSec}[c x]}{i \sqrt{d} \sqrt{e+e x}} + \frac{i \frac{\text{ArcSin}\left[\frac{1}{c x}\right] - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(\sqrt{e+c} + i c \sqrt{d} - \sqrt{-c^2 d e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x}{\sqrt{-c^2 d e} \left(\sqrt{d}-i \sqrt{e} x\right)}\right]}{\sqrt{e}}}{\sqrt{d}} \right)}{16 e^{5/2}} \right) + \left( \frac{7 i \sqrt{d}}{16 e^{5/2}} \left( -\frac{\text{ArcSec}[c x]}{-i \sqrt{d} \sqrt{e+e x}} - \frac{i \frac{\text{ArcSin}\left[\frac{1}{c x}\right] - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(-\sqrt{e+c} + i c \sqrt{d} + \sqrt{-c^2 d e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x}{\sqrt{-c^2 d e} \left(\sqrt{d}+i \sqrt{e} x\right)}\right]}{\sqrt{e}}}{\sqrt{d}} \right)}{16 e^{5/2}} \right) \right)
 \end{aligned}$$

$$\frac{1}{16 e^{5/2}} d \left( -\frac{\text{ArcSec}[c x]}{\sqrt{e} \left(-i \sqrt{d} + \sqrt{e} x\right)^2} + \frac{\frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}} - i \left( \frac{c \sqrt{d} \sqrt{e} \sqrt{1-\frac{1}{c^2 x^2}} x}{(c^2 d+e) \left(-i \sqrt{d} + \sqrt{e} x\right)} + \frac{(2 c^2 d+e) \text{Log}\left[-\frac{4 d \sqrt{e} \sqrt{c^2 d+e} \left(i \sqrt{e} + c \left(c \sqrt{d} - \sqrt{-c^2 d e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{(2 c^2 d+e) \left(-i \sqrt{d} + \sqrt{e} x\right)}\right]}{(c^2 d+e)^{3/2}} \right)}{d} \right) - \frac{1}{16 e^{5/2}}$$

$$\left( \frac{i c \sqrt{e} \sqrt{1-\frac{1}{c^2 x^2}} x}{\sqrt{d} (c^2 d+e) \left(i \sqrt{d} + \sqrt{e} x\right)} - \frac{\text{ArcSec}[c x]}{\sqrt{e} \left(i \sqrt{d} + \sqrt{e} x\right)^2} + \frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{d \sqrt{e}} - \frac{i (2 c^2 d+e) \text{Log}\left[\frac{4 d \sqrt{e} \sqrt{c^2 d+e} \left(-i \sqrt{e} + c \left(c \sqrt{d} + \sqrt{c^2 d+e} \sqrt{1-\frac{1}{c^2 x^2}}\right) x\right)}{(2 c^2 d+e) \left(i \sqrt{d} + \sqrt{e} x\right)}\right]}{d (c^2 d+e)^{3/2}} \right) +$$



$$\begin{aligned}
& \frac{1}{4 e^3} i \left( 8 \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[ \frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec} [c x] \right]}{\sqrt{c^2 d + e}} \right] - 2 i \operatorname{ArcSec} [c x] \operatorname{Log} \left[ 1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] - \right. \\
& 4 i \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] - 2 i \operatorname{ArcSec} [c x] \operatorname{Log} \left[ 1 + \frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] + \\
& 4 i \operatorname{ArcSin} \left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] + 2 i \operatorname{ArcSec} [c x] \operatorname{Log} \left[ 1 + e^{2 i \operatorname{ArcSec} [c x]} \right] - \\
& \left. 2 \operatorname{PolyLog} \left[ 2, \frac{i (-\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] - 2 \operatorname{PolyLog} \left[ 2, -\frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] + \operatorname{PolyLog} \left[ 2, -e^{2 i \operatorname{ArcSec} [c x]} \right] \right) + \\
& \frac{1}{4 e^3} i \left( 8 \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTan} \left[ \frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec} [c x] \right]}{\sqrt{c^2 d + e}} \right] - 2 i \operatorname{ArcSec} [c x] \operatorname{Log} \left[ 1 + \frac{i (-\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] - \right. \\
& 4 i \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i (-\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] - 2 i \operatorname{ArcSec} [c x] \operatorname{Log} \left[ 1 - \frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] + \\
& 4 i \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 - \frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{i \operatorname{ArcSec} [c x]}}{c \sqrt{d}} \right] + 2 i \operatorname{ArcSec} [c x] \operatorname{Log} \left[ 1 + e^{2 i \operatorname{ArcSec} [c x]} \right] - \right.
\end{aligned}$$

$$2 \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right]$$

**Problem 105:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 3, 157 leaves, 6 steps):

$$\frac{b c x \sqrt{-1 + c^2 x^2}}{8 e (c^2 d + e) \sqrt{c^2 x^2} (d + e x^2)} + \frac{x^4 (a + b \operatorname{ArcSec}[c x])}{4 d (d + e x^2)^2} - \frac{b c (c^2 d + 2 e) x \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{\sqrt{c^2 d + e}}\right]}{8 d e^{3/2} (c^2 d + e)^{3/2} \sqrt{c^2 x^2}}$$

Result (type 3, 389 leaves):

$$\begin{aligned}
& -\frac{1}{16e^2} \left( -\frac{4ad}{(d+ex^2)^2} + \frac{8a}{d+ex^2} - \frac{2bce\sqrt{1-\frac{1}{c^2x^2}}x}{(c^2d+e)(d+ex^2)} + \frac{4b(d+2ex^2)\text{ArcSec}[cx]}{(d+ex^2)^2} + \frac{4b\text{ArcSin}\left[\frac{1}{cx}\right]}{d} \right. \\
& \left. + \frac{b\sqrt{e}(c^2d+2e)\text{Log}\left[-\frac{16d\sqrt{-c^2d-e}e^{3/2}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(c^2d+2e)(i\sqrt{d}+\sqrt{e}x)}\right]}{d(-c^2d-e)^{3/2}} + \frac{b\sqrt{e}(c^2d+2e)\text{Log}\left[\frac{16id\sqrt{-c^2d-e}e^{3/2}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(c^2d+2e)(\sqrt{d}+i\sqrt{e}x)}\right]}{d(-c^2d-e)^{3/2}} \right)
\end{aligned}$$

**Problem 106: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x(a+b\text{ArcSec}[cx])}{(d+ex^2)^3} dx$$

Optimal (type 3, 193 leaves, 8 steps):

$$-\frac{bcx\sqrt{-1+c^2x^2}}{8d(c^2d+e)\sqrt{c^2x^2}(d+ex^2)} - \frac{a+b\text{ArcSec}[cx]}{4e(d+ex^2)^2} + \frac{bcx\text{ArcTan}\left[\frac{\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right]}{4d^2e\sqrt{c^2x^2}} - \frac{bc(3c^2d+2e)x\text{ArcTan}\left[\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{\sqrt{c^2d+e}}\right]}{8d^2\sqrt{e}(c^2d+e)^{3/2}\sqrt{c^2x^2}}$$

Result (type 3, 386 leaves):

$$\begin{aligned}
& \frac{1}{16} \left( -\frac{4a}{e(d+ex^2)^2} - \frac{2bc\sqrt{1-\frac{1}{c^2x^2}}x}{d(c^2d+e)(d+ex^2)} - \frac{4b\text{ArcSec}[cx]}{e(d+ex^2)^2} - \frac{4b\text{ArcSin}\left[\frac{1}{cx}\right]}{d^2e} \right. \\
& \left. + \frac{b(3c^2d+2e)\text{Log}\left[-\frac{16d^2\sqrt{-c^2d-e}\sqrt{e}\left(\sqrt{e}+c\left(ic\sqrt{d}-\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)(i\sqrt{d}+\sqrt{e}x)}\right]}{d^2(-c^2d-e)^{3/2}\sqrt{e}} - \frac{b(3c^2d+2e)\text{Log}\left[\frac{16id^2\sqrt{-c^2d-e}\sqrt{e}\left(-\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d-e}\sqrt{1-\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d+2e)(\sqrt{d}+i\sqrt{e}x)}\right]}{d^2(-c^2d-e)^{3/2}\sqrt{e}} \right)
\end{aligned}$$

### Problem 107: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{ArcSec}[c x]}{x (d + e x^2)^3} dx$$

Optimal (type 4, 685 leaves, 28 steps):

$$\begin{aligned} & \frac{b c e \sqrt{1 - \frac{1}{c^2 x^2}}}{8 d^2 (c^2 d + e) \left(e + \frac{d}{x^2}\right) x} + \frac{e^2 (a + b \operatorname{ArcSec}[c x])}{4 d^3 \left(e + \frac{d}{x^2}\right)^2} - \frac{e (a + b \operatorname{ArcSec}[c x])}{d^3 \left(e + \frac{d}{x^2}\right)} + \frac{i (a + b \operatorname{ArcSec}[c x])^2}{2 b d^3} - \\ & \frac{b \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x}\right]}{d^3 \sqrt{c^2 d + e}} + \frac{b \sqrt{e} (c^2 d + 2 e) \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{1 - \frac{1}{c^2 x^2}} x}\right]}{8 d^3 (c^2 d + e)^{3/2}} - \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} - \\ & \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} - \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} - \frac{(a + b \operatorname{ArcSec}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} + \\ & \frac{i b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} + \frac{i b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} + \frac{i b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} + \frac{i b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{i \operatorname{ArcSec}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} \end{aligned}$$

Result (type 4, 1871 leaves):

$$\frac{a}{4 d (d + e x^2)^2} + \frac{a}{2 d^2 (d + e x^2)} + \frac{a \operatorname{Log}[x]}{d^3} - \frac{a \operatorname{Log}[d + e x^2]}{2 d^3} +$$

$$b \left[ \frac{5 i \sqrt{e}}{16 d^{5/2}} \left( -\frac{\text{ArcSec}[c x]}{i \sqrt{d} \sqrt{e+e x}} + \frac{i \frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}} - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(\sqrt{e+c} \left(i c \sqrt{d}-\sqrt{-c^2 d+e}\right) \sqrt{1-\frac{1}{c^2 x^2}}\right) x}{\sqrt{-c^2 d+e} \left(\sqrt{d}-i \sqrt{e} x\right)}\right]}{\sqrt{d}}}{\sqrt{d}} \right) + \frac{5 i \sqrt{e}}{16 d^{5/2}} \left( -\frac{\text{ArcSec}[c x]}{-i \sqrt{d} \sqrt{e+e x}} - \frac{i \frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}} - \frac{\text{Log}\left[\frac{2 \sqrt{d} \sqrt{e} \left(-\sqrt{e+c} \left(i c \sqrt{d}+\sqrt{-c^2 d+e}\right) \sqrt{1-\frac{1}{c^2 x^2}}\right) x}{\sqrt{-c^2 d+e} \left(\sqrt{d}+i \sqrt{e} x\right)}\right]}{\sqrt{d}}}{\sqrt{d}} \right) \right]$$

$$\frac{1}{16 d^2} \sqrt{e} \left( -\frac{\text{ArcSec}[c x]}{\sqrt{e} \left(-i \sqrt{d} + \sqrt{e} x\right)^2} + \frac{\frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{\sqrt{e}} - i \left( \frac{c \sqrt{d} \sqrt{e} \sqrt{1-\frac{1}{c^2 x^2}} x}{(c^2 d+e) \left(-i \sqrt{d} + \sqrt{e} x\right)} + \frac{(2 c^2 d+e) \text{Log}\left[\frac{4 d \sqrt{e} \sqrt{c^2 d+e} \left(i \sqrt{e+c} \left(c \sqrt{d}-\sqrt{-c^2 d+e}\right) \sqrt{1-\frac{1}{c^2 x^2}}\right) x}{(2 c^2 d+e) \left(-i \sqrt{d} + \sqrt{e} x\right)}\right]}{(c^2 d+e)^{3/2}} \right)}{d} \right) + \frac{1}{16 d^2}$$

$$\sqrt{e} \left( \frac{i c \sqrt{e} \sqrt{1-\frac{1}{c^2 x^2}} x}{\sqrt{d} (c^2 d+e) \left(i \sqrt{d} + \sqrt{e} x\right)} - \frac{\text{ArcSec}[c x]}{\sqrt{e} \left(i \sqrt{d} + \sqrt{e} x\right)^2} + \frac{\text{ArcSin}\left[\frac{1}{c x}\right]}{d \sqrt{e}} - \frac{i (2 c^2 d+e) \text{Log}\left[\frac{4 d \sqrt{e} \sqrt{c^2 d+e} \left(-i \sqrt{e+c} \left(c \sqrt{d} + \sqrt{c^2 d+e}\right) \sqrt{1-\frac{1}{c^2 x^2}}\right) x}{(2 c^2 d+e) \left(i \sqrt{d} + \sqrt{e} x\right)}\right]}{d (c^2 d+e)^{3/2}} \right) +$$

$$\begin{aligned}
& \frac{\frac{1}{2} i \operatorname{ArcSec}[c x]^2 - \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSec}[c x]}\right] + \frac{1}{2} i \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right]}{d^3} - \\
& \frac{1}{4 d^3} i \left( 8 \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(i c \sqrt{d} + \sqrt{e}\right) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[c x]\right]}{\sqrt{c^2 d + e}}\right] - 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - \right. \\
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \\
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSec}[c x]}\right] - \\
& \left. 2 \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right] \right) - \\
& \frac{1}{4 d^3} i \left( 8 \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTan}\left[\frac{\left(-i c \sqrt{d} + \sqrt{e}\right) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[c x]\right]}{\sqrt{c^2 d + e}}\right] - 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - \right. \\
& 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \\
& \left. 4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + 2 i \operatorname{ArcSec}[c x] \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSec}[c x]}\right] - \right.
\end{aligned}$$

$$2 \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] - 2 \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{i \operatorname{ArcSec}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[c x]}\right]$$

**Problem 111: Result unnecessarily involves higher level functions.**

$$\int x^5 \sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x]) dx$$

Optimal (type 3, 403 leaves, 12 steps):

$$\frac{b (23 c^4 d^2 + 12 c^2 d e - 75 e^2) x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{1680 c^5 e^2 \sqrt{c^2 x^2}} + \frac{b (29 c^2 d - 25 e) x \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{840 c^3 e^2 \sqrt{c^2 x^2}} -$$

$$\frac{b x \sqrt{-1 + c^2 x^2} (d + e x^2)^{5/2}}{42 c e^2 \sqrt{c^2 x^2}} + \frac{d^2 (d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 e^3} - \frac{2 d (d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{5 e^3} +$$

$$\frac{(d + e x^2)^{7/2} (a + b \operatorname{ArcSec}[c x])}{7 e^3} + \frac{8 b c d^{7/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{105 e^3 \sqrt{c^2 x^2}} - \frac{b (105 c^6 d^3 - 35 c^4 d^2 e + 63 c^2 d e^2 + 75 e^3) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}}\right]}{1680 c^6 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 706 leaves):

$$\begin{aligned}
& - \left( \left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \left( (105 c^6 d^3 - 35 c^4 d^2 e + 63 c^2 d e^2 + 75 e^3) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d} \right] \right. \right. \right. \\
& \quad \left( c^2 d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] - e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) + 4 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \\
& \quad \left. \left( (35 c^6 d^2 e^2 x^2 - 63 c^4 d e^3 x^2 - 75 c^2 e^4 x^2 + c^8 d^3 (128 d - 105 e x^2)) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d} \right] + \right. \right. \\
& \quad \left. \left. 32 c^8 d^3 x^2 \left( -e \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] + c^2 d \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] \right) \right) \right) \right) / \\
& \quad \left( 840 c^5 e^2 (-1 + c^2 x^2) \sqrt{d + e x^2} \left( -4 c^2 e x^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] + c^2 d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] - \right. \right. \\
& \quad \left. \left. e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) \right) \\
& \quad \left. \left( 4 d \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d} \right] + x^2 \left( -e \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] + c^2 d \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] \right) \right) \right) \right) + \\
& \quad \frac{1}{1680 c^5 e^3} \sqrt{d + e x^2} \left( 16 a c^5 (8 d^3 - 4 d^2 e x^2 + 3 d e^2 x^4 + 15 e^3 x^6) - b e \sqrt{1 - \frac{1}{c^2 x^2}} x (75 e^2 + 2 c^2 e (19 d + 25 e x^2) + c^4 (-41 d^2 + 22 d e x^2 + 40 e^2 x^4)) + \right. \\
& \quad \left. 16 b c^5 (8 d^3 - 4 d^2 e x^2 + 3 d e^2 x^4 + 15 e^3 x^6) \operatorname{ArcSec}[c x] \right)
\end{aligned}$$

**Problem 112:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x]) dx$$

Optimal (type 3, 294 leaves, 11 steps):

$$\begin{aligned}
& - \frac{b (c^2 d + 9 e) x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{120 c^3 e \sqrt{c^2 x^2}} - \frac{b x \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{20 c e \sqrt{c^2 x^2}} - \frac{d (d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 e^2} + \\
& \frac{(d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{5 e^2} - \frac{2 b c d^{5/2} x \operatorname{ArcTan} \left[ \frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}} \right]}{15 e^2 \sqrt{c^2 x^2}} + \frac{b (15 c^4 d^2 - 10 c^2 d e - 9 e^2) x \operatorname{ArcTanh} \left[ \frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}} \right]}{120 c^4 e^{3/2} \sqrt{c^2 x^2}}
\end{aligned}$$



Result (type 6, 628 leaves):

$$\left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \left( (15 c^4 d^2 - 10 c^2 d e - 9 e^2) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \right. \right. \\ \left. \left. \left( c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) + \right. \\ \left. 4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left( (10 c^4 d e^2 x^2 + 9 c^2 e^3 x^2 + c^6 d^2 (16 d - 15 e x^2)) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + \right. \right. \\ \left. \left. 4 c^6 d^2 x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Bigg/ \\ \left( 60 c^3 e (-1 + c^2 x^2) \sqrt{d + e x^2} \left( -4 c^2 e x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - \right. \right. \\ \left. \left. e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) \right. \\ \left. \left( 4 d \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Bigg) + \\ \frac{1}{120 c^3 e^2} \sqrt{d + e x^2} \left( 8 a c^3 (-2 d^2 + d e x^2 + 3 e^2 x^4) - b e \sqrt{1 - \frac{1}{c^2 x^2}} x (9 e + c^2 (7 d + 6 e x^2)) + 8 b c^3 (-2 d^2 + d e x^2 + 3 e^2 x^4) \operatorname{ArcSec}[c x] \right)$$

**Problem 113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x \sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x]) dx$$

Optimal (type 3, 195 leaves, 9 steps):

$$-\frac{b x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{6 c \sqrt{c^2 x^2}} + \frac{(d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 e} + \frac{b c d^{3/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{3 e \sqrt{c^2 x^2}} - \frac{b (3 c^2 d + e) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}}\right]}{6 c^2 \sqrt{e} \sqrt{c^2 x^2}}$$

Result (type 6, 548 leaves):

$$\begin{aligned}
& - \left( \left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \right. \right. \\
& \quad \left( (3 c^2 d + e) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d} \right] \left( c^2 d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] - e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) + \right. \\
& \quad \left. 2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \left( (-2 c^2 e^2 x^2 + 2 c^4 d (2 d - 3 e x^2)) \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d} \right] + \right. \right. \\
& \quad \left. \left. c^4 d x^2 \left( -e \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] + c^2 d \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] \right) \right) \right) \Bigg/ \left( 3 c (-1 + c^2 x^2) \sqrt{d + e x^2} \right. \\
& \quad \left. \left( -4 c^2 e x^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] + c^2 d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] - e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) \right. \\
& \quad \left. \left( 4 d \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d} \right] + x^2 \left( -e \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] + c^2 d \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d} \right] \right) \right) \right) \Bigg) + \\
& \quad \frac{\sqrt{d + e x^2} \left( -b e \sqrt{1 - \frac{1}{c^2 x^2}} x + 2 a c (d + e x^2) + 2 b c (d + e x^2) \operatorname{ArcSec}[c x] \right)}{6 c e}
\end{aligned}$$

**Problem 119:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x])}{x^4} dx$$

Optimal (type 4, 328 leaves, 11 steps):

$$\begin{aligned}
& \frac{2 b c (c^2 d + 2 e) \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{9 d \sqrt{c^2 x^2}} + \frac{b c \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{9 x^2 \sqrt{c^2 x^2}} - \\
& \frac{(d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 d x^3} - \frac{2 b c^2 (c^2 d + 2 e) x \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{9 d \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{1 + \frac{e x^2}{d}}} + \\
& \frac{b (c^2 d + e) (2 c^2 d + 3 e) x \sqrt{1 - c^2 x^2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{9 d \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}
\end{aligned}$$

Result (type 4, 247 leaves):

$$\frac{\sqrt{d+ex^2} \left( -3a(d+ex^2) + bc \sqrt{1 - \frac{1}{c^2 x^2}} x (d + 2c^2 dx^2 + 4ex^2) - 3b(d+ex^2) \operatorname{ArcSec}[cx] \right)}{9dx^3} -$$

$$\left( ic \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{1 + \frac{ex^2}{d}} \left( 2c^2 d (c^2 d + 2e) \operatorname{EllipticE} \left[ i \operatorname{ArcSinh} \left[ \sqrt{-c^2} x \right], -\frac{e}{c^2 d} \right] - \right. \right.$$

$$\left. \left. (2c^4 d^2 + 5c^2 de + 3e^2) \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \sqrt{-c^2} x \right], -\frac{e}{c^2 d} \right] \right) \right) / \left( 9 \sqrt{-c^2} d \sqrt{1 - c^2 x^2} \sqrt{d + ex^2} \right)$$

**Problem 120: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{d+ex^2} (a + b \operatorname{ArcSec}[cx])}{x^6} dx$$

Optimal (type 4, 453 leaves, 12 steps):

$$\frac{bc (24c^4 d^2 + 19c^2 de - 31e^2) \sqrt{-1+c^2 x^2} \sqrt{d+ex^2}}{225d^2 \sqrt{c^2 x^2}} + \frac{bc (12c^2 d - e) \sqrt{-1+c^2 x^2} \sqrt{d+ex^2}}{225dx^2 \sqrt{c^2 x^2}} +$$

$$\frac{bc \sqrt{-1+c^2 x^2} (d+ex^2)^{3/2}}{25dx^4 \sqrt{c^2 x^2}} - \frac{(d+ex^2)^{3/2} (a + b \operatorname{ArcSec}[cx])}{5dx^5} + \frac{2e (d+ex^2)^{3/2} (a + b \operatorname{ArcSec}[cx])}{15d^2 x^3} -$$

$$\frac{bc^2 (24c^4 d^2 + 19c^2 de - 31e^2) x \sqrt{1-c^2 x^2} \sqrt{d+ex^2} \operatorname{EllipticE} \left[ \operatorname{ArcSin}[cx], -\frac{e}{c^2 d} \right]}{225d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1 + \frac{ex^2}{d}}} +$$

$$\frac{b (c^2 d + e) (24c^4 d^2 + 7c^2 de - 30e^2) x \sqrt{1-c^2 x^2} \sqrt{1 + \frac{ex^2}{d}} \operatorname{EllipticF} \left[ \operatorname{ArcSin}[cx], -\frac{e}{c^2 d} \right]}{225d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+ex^2}}$$

Result (type 4, 325 leaves):

$$\frac{1}{225 d^2 x^5} \sqrt{d + e x^2} \left( -15 a (3 d^2 + d e x^2 - 2 e^2 x^4) + \right.$$

$$\left. b c \sqrt{1 - \frac{1}{c^2 x^2}} x (-31 e^2 x^4 + d e x^2 (8 + 19 c^2 x^2) + 3 d^2 (3 + 4 c^2 x^2 + 8 c^4 x^4)) - 15 b (3 d^2 + d e x^2 - 2 e^2 x^4) \operatorname{ArcSec}[c x] \right) -$$

$$\left( i b c \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{1 + \frac{e x^2}{d}} \left( c^2 d (24 c^4 d^2 + 19 c^2 d e - 31 e^2) \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-c^2} x \right], -\frac{e}{c^2 d} \right] + \right. \right.$$

$$\left. \left. (-24 c^6 d^3 - 31 c^4 d^2 e + 23 c^2 d e^2 + 30 e^3) \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \sqrt{-c^2} x \right], -\frac{e}{c^2 d} \right] \right) \right) / \left( 225 \sqrt{-c^2} d^2 \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \right)$$

**Problem 121: Result unnecessarily involves higher level functions.**

$$\int x^3 (d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x]) dx$$

Optimal (type 3, 374 leaves, 12 steps):

$$\frac{b (3 c^4 d^2 - 38 c^2 d e - 25 e^2) x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{560 c^5 e \sqrt{c^2 x^2}} - \frac{b (13 c^2 d + 25 e) x \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{840 c^3 e \sqrt{c^2 x^2}} -$$

$$\frac{b x \sqrt{-1 + c^2 x^2} (d + e x^2)^{5/2}}{42 c e \sqrt{c^2 x^2}} - \frac{d (d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{5 e^2} + \frac{(d + e x^2)^{7/2} (a + b \operatorname{ArcSec}[c x])}{7 e^2} -$$

$$\frac{2 b c d^{7/2} x \operatorname{ArcTan}\left[ \frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}} \right]}{35 e^2 \sqrt{c^2 x^2}} + \frac{b (35 c^6 d^3 - 35 c^4 d^2 e - 63 c^2 d e^2 - 25 e^3) x \operatorname{ArcTanh}\left[ \frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}} \right]}{560 c^6 e^{3/2} \sqrt{c^2 x^2}}$$

Result (type 6, 679 leaves):

$$\begin{aligned}
& \left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \left( (35 c^6 d^3 - 35 c^4 d^2 e - 63 c^2 d e^2 - 25 e^3) \right. \right. \\
& \quad \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \left( c^2 d \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) + \\
& \quad 4 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left( (35 c^6 d^2 e^2 x^2 + 63 c^4 d e^3 x^2 + 25 c^2 e^4 x^2 + c^8 d^3 (32 d - 35 e x^2)) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + \right. \\
& \quad \left. 8 c^8 d^3 x^2 \left( -e \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \Bigg) / \\
& \quad \left( 280 c^5 e (-1 + c^2 x^2) \sqrt{d + e x^2} \left( -4 c^2 e x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - \right. \right. \\
& \quad \left. \left. e \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) \right) \\
& \quad \left( 4 d \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \Bigg) - \\
& \quad \frac{1}{1680 c^5 e^2} \sqrt{d + e x^2} \left( 48 a c^5 (2 d - 5 e x^2) (d + e x^2)^2 + b e \sqrt{1 - \frac{1}{c^2 x^2}} x (75 e^2 + 2 c^2 e (82 d + 25 e x^2) + c^4 (57 d^2 + 106 d e x^2 + 40 e^2 x^4)) + \right. \\
& \quad \left. 48 b c^5 (2 d - 5 e x^2) (d + e x^2)^2 \text{ArcSec}[c x] \right)
\end{aligned}$$

**Problem 122: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int x (d + e x^2)^{3/2} (a + b \text{ArcSec}[c x]) dx$$

Optimal (type 3, 262 leaves, 10 steps):

$$\begin{aligned}
& - \frac{b (7 c^2 d + 3 e) x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{40 c^3 \sqrt{c^2 x^2}} - \frac{b x \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{20 c \sqrt{c^2 x^2}} + \\
& \frac{(d + e x^2)^{5/2} (a + b \text{ArcSec}[c x])}{5 e} + \frac{b c d^{5/2} x \text{ArcTan}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{5 e \sqrt{c^2 x^2}} - \frac{b (15 c^4 d^2 + 10 c^2 d e + 3 e^2) x \text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}}\right]}{40 c^4 \sqrt{e} \sqrt{c^2 x^2}}
\end{aligned}$$

Result (type 6, 604 leaves):

$$\begin{aligned}
 & \left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \left( - (15 c^4 d^2 + 10 c^2 d e + 3 e^2) \right. \right. \\
 & \quad \left. \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \left( c^2 d \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) + \right. \\
 & \quad \left. \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left( (40 c^4 d e^2 x^2 + 12 c^2 e^3 x^2 + 4 c^6 d^2 (-8 d + 15 e x^2)) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + \right. \right. \\
 & \quad \left. \left. 8 c^6 d^2 x^2 \left( e \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] - c^2 d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Bigg/ \\
 & \left( 20 c^3 (-1 + c^2 x^2) \sqrt{d + e x^2} \left( -4 c^2 e x^2 \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - \right. \right. \\
 & \quad \left. \left. e \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) \right. \\
 & \quad \left. \left( 4 d \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Bigg) + \\
 & \frac{\sqrt{d + e x^2} \left( 8 a c^3 (d + e x^2)^2 - b e \sqrt{1 - \frac{1}{c^2 x^2}} x (3 e + c^2 (9 d + 2 e x^2)) + 8 b c^3 (d + e x^2)^2 \text{ArcSec}[c x] \right)}{40 c^3 e}
 \end{aligned}$$

**Problem 129: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d + e x^2)^{3/2} (a + b \text{ArcSec}[c x])}{x^6} dx$$

Optimal (type 4, 416 leaves, 12 steps):

$$\frac{b c (8 c^4 d^2 + 23 c^2 d e + 23 e^2) \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{75 d \sqrt{c^2 x^2}} + \frac{4 b c (c^2 d + 2 e) \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{75 x^2 \sqrt{c^2 x^2}} + \frac{b c \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{25 x^4 \sqrt{c^2 x^2}} -$$

$$\frac{(d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{5 d x^5} - \frac{b c^2 (8 c^4 d^2 + 23 c^2 d e + 23 e^2) x \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{75 d \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{1 + \frac{e x^2}{d}}} +$$

$$\frac{b (c^2 d + e) (8 c^4 d^2 + 19 c^2 d e + 15 e^2) x \sqrt{1 - c^2 x^2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{75 d \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}$$

Result (type 4, 303 leaves):

$$\frac{1}{75 d x^5} \sqrt{d + e x^2} \left( -15 a (d + e x^2)^2 + b c \sqrt{1 - \frac{1}{c^2 x^2}} x (23 e^2 x^4 + d e x^2 (11 + 23 c^2 x^2) + d^2 (3 + 4 c^2 x^2 + 8 c^4 x^4)) - 15 b (d + e x^2)^2 \operatorname{ArcSec}[c x] \right) -$$

$$\left( i b c \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{1 + \frac{e x^2}{d}} \left( c^2 d (8 c^4 d^2 + 23 c^2 d e + 23 e^2) \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] - \right. \right.$$

$$\left. \left. (8 c^6 d^3 + 27 c^4 d^2 e + 34 c^2 d e^2 + 15 e^3) \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] \right) \right) / \left( 75 \sqrt{-c^2} d \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \right)$$

**Problem 130: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{x^8} dx$$

Optimal (type 4, 554 leaves, 13 steps):

$$\frac{b c (240 c^6 d^3 + 528 c^4 d^2 e + 193 c^2 d e^2 - 247 e^3) \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{3675 d^2 \sqrt{c^2 x^2}} + \frac{b c (120 c^4 d^2 + 159 c^2 d e - 37 e^2) \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{3675 d x^2 \sqrt{c^2 x^2}} +$$

$$\frac{b c (30 c^2 d + 11 e) \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{1225 d x^4 \sqrt{c^2 x^2}} + \frac{b c \sqrt{-1 + c^2 x^2} (d + e x^2)^{5/2}}{49 d x^6 \sqrt{c^2 x^2}} - \frac{(d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{7 d x^7} +$$

$$\frac{2 e (d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{35 d^2 x^5} - \frac{b c^2 (240 c^6 d^3 + 528 c^4 d^2 e + 193 c^2 d e^2 - 247 e^3) x \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3675 d^2 \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{1 + \frac{e x^2}{d}}} +$$

$$\left( 2 b (c^2 d + e) (120 c^6 d^3 + 204 c^4 d^2 e + 17 c^2 d e^2 - 105 e^3) x \sqrt{1 - c^2 x^2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}] \right) /$$

$$\left( 3675 d^2 \sqrt{c^2 x^2} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2} \right)$$

Result (type 4, 383 leaves):

$$\frac{1}{3675 d^2 x^7} \sqrt{d + e x^2} \left( -105 a (5 d - 2 e x^2) (d + e x^2)^2 + \right.$$

$$b c \sqrt{1 - \frac{1}{c^2 x^2}} x (-247 e^3 x^6 + d e^2 x^4 (71 + 193 c^2 x^2) + 3 d^2 e x^2 (61 + 83 c^2 x^2 + 176 c^4 x^4) + 15 d^3 (5 + 6 c^2 x^2 + 8 c^4 x^4 + 16 c^6 x^6)) -$$

$$\left. 105 b (5 d - 2 e x^2) (d + e x^2)^2 \operatorname{ArcSec}[c x] \right) -$$

$$\left( i b c \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{1 + \frac{e x^2}{d}} \left( c^2 d (240 c^6 d^3 + 528 c^4 d^2 e + 193 c^2 d e^2 - 247 e^3) \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] - \right. \right.$$

$$\left. \left. 2 (120 c^8 d^4 + 324 c^6 d^3 e + 221 c^4 d^2 e^2 - 88 c^2 d e^3 - 105 e^4) \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] \right) \right) / \left( 3675 \sqrt{-c^2} d^2 \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \right)$$

**Problem 131:** Result unnecessarily involves higher level functions.

$$\int \frac{x^5 (a + b \operatorname{ArcSec}[c x])}{\sqrt{d + e x^2}} dx$$



Optimal (type 3, 321 leaves, 11 steps):

$$\frac{b (19 c^2 d - 9 e) x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{120 c^3 e^2 \sqrt{c^2 x^2}} - \frac{b x \sqrt{-1 + c^2 x^2} (d + e x^2)^{3/2}}{20 c e^2 \sqrt{c^2 x^2}} + \frac{d^2 \sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x])}{e^3} - \frac{2 d (d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 e^3} +$$

$$\frac{(d + e x^2)^{5/2} (a + b \operatorname{ArcSec}[c x])}{5 e^3} + \frac{8 b c d^{5/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{15 e^3 \sqrt{c^2 x^2}} - \frac{b (45 c^4 d^2 - 10 c^2 d e + 9 e^2) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}}\right]}{120 c^4 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 629 leaves):

$$- \left( \left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \left( (45 c^4 d^2 - 10 c^2 d e + 9 e^2) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \right. \right. \right.$$

$$\left. \left. \left( c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) + \right. \right.$$

$$\left. 4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left( (10 c^4 d e x^2 - 9 c^2 e^3 x^2 + c^6 d^2 (64 d - 45 e x^2)) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + \right. \right.$$

$$\left. \left. 16 c^6 d^2 x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Big/ \left( 60 c^3 e^2 (-1 + c^2 x^2) \sqrt{d + e x^2} \right.$$

$$\left. \left( -4 c^2 e x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) \right.$$

$$\left. \left. \left( 4 d \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \right) +$$

$$\frac{1}{120 c^3 e^3} \sqrt{d + e x^2} \left( 8 a c^3 (8 d^2 - 4 d e x^2 + 3 e^2 x^4) + b e \sqrt{1 - \frac{1}{c^2 x^2}} x (-9 e + c^2 (13 d - 6 e x^2)) + \right.$$

$$\left. 8 b c^3 (8 d^2 - 4 d e x^2 + 3 e^2 x^4) \operatorname{ArcSec}[c x] \right)$$

**Problem 132:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a + b \operatorname{ArcSec}[c x])}{\sqrt{d + e x^2}} dx$$

Optimal (type 3, 225 leaves, 10 steps):

$$\frac{b x \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{6 c e \sqrt{c^2 x^2}} - \frac{d \sqrt{d+e x^2} (a+b \operatorname{ArcSec}[c x])}{e^2} +$$

$$\frac{(d+e x^2)^{3/2} (a+b \operatorname{ArcSec}[c x])}{3 e^2} - \frac{2 b c d^{3/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{3 e^2 \sqrt{c^2 x^2}} + \frac{b (3 c^2 d - e) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{6 c^2 e^{3/2} \sqrt{c^2 x^2}}$$

Result (type 6, 555 leaves):

$$\left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \right.$$

$$\left( (3 c^2 d - e) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \left( c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) + \right.$$

$$4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left( (c^2 e^2 x^2 + c^4 d (4 d - 3 e x^2)) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + \right.$$

$$\left. \left. c^4 d x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) / \left( 3 c e (-1 + c^2 x^2) \sqrt{d+e x^2} \right.$$

$$\left. \left( -4 c^2 e x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) \right.$$

$$\left. \left( 4 d \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) -$$

$$\frac{\sqrt{d+e x^2} \left( 4 a c d + b e \sqrt{1 - \frac{1}{c^2 x^2}} x - 2 a c e x^2 + 2 b c (2 d - e x^2) \operatorname{ArcSec}[c x] \right)}{6 c e^2}$$

**Problem 133: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{x (a + b \operatorname{ArcSec}[c x])}{\sqrt{d+e x^2}} dx$$

Optimal (type 3, 132 leaves, 9 steps):

$$\frac{\sqrt{d+ex^2} (a+b \operatorname{ArcSec}[cx])}{e} + \frac{bc\sqrt{d} x \operatorname{ArcTan}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right]}{e\sqrt{c^2x^2}} - \frac{bx \operatorname{ArcTanh}\left[\frac{\sqrt{e}\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right]}{\sqrt{e}\sqrt{c^2x^2}}$$

Result (type 6, 271 leaves):

$$\left( 3b(c^2d+e) \sqrt{1-\frac{1}{c^2x^2}} \sqrt{d+ex^2} \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{e-c^2ex^2}{c^2d+e}, 1-c^2x^2\right] \right) /$$

$$\left( cex \left( -3(c^2d+e) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, 1, \frac{3}{2}, \frac{e-c^2ex^2}{c^2d+e}, 1-c^2x^2\right] + (-1+c^2x^2) \left( 2(c^2d+e) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}, \frac{e-c^2ex^2}{c^2d+e}, 1-c^2x^2\right] - \right. \right. \right.$$

$$\left. \left. \left. e \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{e-c^2ex^2}{c^2d+e}, 1-c^2x^2\right] \right) \right) \right) + \frac{\sqrt{d+ex^2} (a+b \operatorname{ArcSec}[cx])}{e}$$

**Problem 139: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSec}[cx]}{x^4 \sqrt{d+ex^2}} dx$$

Optimal (type 4, 362 leaves, 11 steps):

$$\frac{bc(2c^2d-5e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{c^2x^2}} - \frac{\sqrt{d+ex^2}(a+b \operatorname{ArcSec}[cx])}{3dx^3} +$$

$$\frac{2e\sqrt{d+ex^2}(a+b \operatorname{ArcSec}[cx])}{3d^2x} - \frac{bc^2(2c^2d-5e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2} \operatorname{EllipticE}[\operatorname{ArcSin}[cx], -\frac{e}{c^2d}]}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} +$$

$$\frac{2b(c^2d-3e)(c^2d+e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[cx], -\frac{e}{c^2d}]}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}$$

Result (type 4, 249 leaves):

$$\frac{\sqrt{d+ex^2} \left( bc \sqrt{1-\frac{1}{c^2x^2}} x (d+2c^2dx^2-5ex^2) - 3a(d-2ex^2) - 3b(d-2ex^2) \operatorname{ArcSec}[cx] \right)}{9d^2x^3} -$$

$$\left( i b c \sqrt{1-\frac{1}{c^2x^2}} x \sqrt{1+\frac{ex^2}{d}} \left( c^2d(2c^2d-5e) \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[\sqrt{-c^2}x\right], -\frac{e}{c^2d} \right] + \right. \right.$$

$$\left. \left. 2(-c^4d^2+2c^2de+3e^2) \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{-c^2}x\right], -\frac{e}{c^2d} \right] \right) \right) / \left( 9\sqrt{-c^2}d^2\sqrt{1-c^2x^2}\sqrt{d+ex^2} \right)$$

**Problem 140:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a + b \operatorname{ArcSec}[cx]}{x^6 \sqrt{d+ex^2}} dx$$

Optimal (type 4, 1006 leaves, 32 steps):

$$\begin{aligned}
& \frac{8 b c e^2 \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{15 d^3 \sqrt{c^2 x^2}} - \frac{4 b c e (2 c^2 d+e) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{45 d^3 \sqrt{c^2 x^2}} + \frac{b c (8 c^4 d^2+3 c^2 d e-2 e^2) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{75 d^3 \sqrt{c^2 x^2}} + \\
& \frac{b c \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{25 d x^4 \sqrt{c^2 x^2}} - \frac{4 b c e \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{45 d^2 x^2 \sqrt{c^2 x^2}} + \frac{b c (4 c^2 d+e) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{75 d^2 x^2 \sqrt{c^2 x^2}} - \\
& \frac{\sqrt{d+e x^2} (a+b \operatorname{ArcSec}[c x])}{5 d x^5} + \frac{4 e \sqrt{d+e x^2} (a+b \operatorname{ArcSec}[c x])}{15 d^2 x^3} - \frac{8 e^2 \sqrt{d+e x^2} (a+b \operatorname{ArcSec}[c x])}{15 d^3 x} - \\
& \frac{8 b c^2 e^2 x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{15 d^3 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}} + \frac{4 b c^2 e (2 c^2 d+e) x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{45 d^3 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}} - \\
& \frac{b c^2 (8 c^4 d^2+3 c^2 d e-2 e^2) x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{75 d^3 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}} + \\
& \frac{b c^2 (8 c^2 d-e) (c^2 d+e) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{75 d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}} - \\
& \frac{8 b c^2 e (c^2 d+e) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{45 d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}} + \frac{8 b e^2 (c^2 d+e) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{15 d^3 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}
\end{aligned}$$

Result (type 4, 329 leaves):

$$\frac{1}{225 d^3 x^5} \sqrt{d + e x^2} \left( -15 a (3 d^2 - 4 d e x^2 + 8 e^2 x^4) + \right.$$

$$\left. b c \sqrt{1 - \frac{1}{c^2 x^2}} x (94 e^2 x^4 - d e x^2 (17 + 31 c^2 x^2) + 3 d^2 (3 + 4 c^2 x^2 + 8 c^4 x^4)) - 15 b (3 d^2 - 4 d e x^2 + 8 e^2 x^4) \operatorname{ArcSec}[c x] \right) -$$

$$\left( i b c \sqrt{1 - \frac{1}{c^2 x^2}} x \sqrt{1 + \frac{e x^2}{d}} \left( c^2 d (24 c^4 d^2 - 31 c^2 d e + 94 e^2) \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] - \right. \right.$$

$$\left. \left. (24 c^6 d^3 - 19 c^4 d^2 e + 77 c^2 d e^2 + 120 e^3) \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] \right) \right) / \left( 225 \sqrt{-c^2} d^3 \sqrt{1 - c^2 x^2} \sqrt{d + e x^2} \right)$$

**Problem 141:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^{3/2}} dx$$

Optimal (type 3, 252 leaves, 10 steps):

$$-\frac{b x \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{6 c e^2 \sqrt{c^2 x^2}} - \frac{d^2 (a + b \operatorname{ArcSec}[c x])}{e^3 \sqrt{d + e x^2}} - \frac{2 d \sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x])}{e^3} +$$

$$\frac{(d + e x^2)^{3/2} (a + b \operatorname{ArcSec}[c x])}{3 e^3} - \frac{8 b c d^{3/2} x \operatorname{ArcTan}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{3 e^3 \sqrt{c^2 x^2}} + \frac{b (9 c^2 d - e) x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}}\right]}{6 c^2 e^{5/2} \sqrt{c^2 x^2}}$$

Result (type 6, 587 leaves):

$$\begin{aligned}
& \left( b d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \right. \\
& \left( (9 c^2 d - e) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \left( c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) + \right. \\
& 4 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left( (c^2 e^2 x^2 + c^4 d (16 d - 9 e x^2)) \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + \right. \\
& \left. \left. 4 c^4 d x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Bigg/ \left( 3 c e^2 (-1 + c^2 x^2) \sqrt{d + e x^2} \right. \\
& \left. \left( -4 c^2 e x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) \right. \\
& \left. \left( 4 d \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) \Bigg) + \\
& \left. - b e \sqrt{1 - \frac{1}{c^2 x^2}} x (d + e x^2) - 2 a c (8 d^2 + 4 d e x^2 - e^2 x^4) - 2 b c (8 d^2 + 4 d e x^2 - e^2 x^4) \operatorname{ArcSec}[c x] \right) \\
& \hline
& 6 c e^3 \sqrt{d + e x^2}
\end{aligned}$$

**Problem 142: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{x^3 (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^{3/2}} dx$$

Optimal (type 3, 157 leaves, 9 steps):

$$\frac{d (a + b \operatorname{ArcSec}[c x])}{e^2 \sqrt{d + e x^2}} + \frac{\sqrt{d + e x^2} (a + b \operatorname{ArcSec}[c x])}{e^2} + \frac{2 b c \sqrt{d} x \operatorname{ArcTan}\left[\frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}}\right]}{e^2 \sqrt{c^2 x^2}} - \frac{b x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1 + c^2 x^2}}{c \sqrt{d + e x^2}}\right]}{e^{3/2} \sqrt{c^2 x^2}}$$

Result (type 6, 326 leaves):

$$\begin{aligned}
& - \frac{1}{e(-1+c^2x^2)\sqrt{d+ex^2}} 2bcd \sqrt{1-\frac{1}{c^2x^2}} x^3 \\
& \left( - \left( \left( 2c^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] \right) / \left( 4c^2ex^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] - c^2d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] \right) + \right. \\
& \quad \left. e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] \right) + \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2x^2, -\frac{ex^2}{d} \right] / \left( 4d \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, c^2x^2, -\frac{ex^2}{d} \right] + \right. \\
& \quad \left. x^2 \left( -e \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, c^2x^2, -\frac{ex^2}{d} \right] + c^2d \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, c^2x^2, -\frac{ex^2}{d} \right] \right) \right) + \frac{(2d+ex^2)(a+b \operatorname{ArcSec}[cx])}{e^2\sqrt{d+ex^2}}
\end{aligned}$$

**Problem 143:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x(a+b \operatorname{ArcSec}[cx])}{(d+ex^2)^{3/2}} dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$\frac{a+b \operatorname{ArcSec}[cx]}{e\sqrt{d+ex^2}} - \frac{bcx \operatorname{ArcTan}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1+c^2x^2}}\right]}{\sqrt{d}e\sqrt{c^2x^2}}$$

Result (type 6, 190 leaves):

$$\begin{aligned}
& - \left( \left( 2bc^3 \sqrt{1-\frac{1}{c^2x^2}} x^3 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] \right) / \left( (-1+c^2x^2)\sqrt{d+ex^2} \left( 4c^2ex^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] - \right. \right. \right. \\
& \quad \left. \left. c^2d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] + e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2x^2}, -\frac{d}{ex^2} \right] \right) \right) - \frac{a+b \operatorname{ArcSec}[cx]}{e\sqrt{d+ex^2}}
\end{aligned}$$

**Problem 149:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \operatorname{ArcSec}[cx]}{x^2(d+ex^2)^{3/2}} dx$$

Optimal (type 4, 274 leaves, 10 steps):



$$\frac{b c \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{d^2 \sqrt{c^2 x^2}} - \frac{a+b \operatorname{ArcSec}[c x]}{d x \sqrt{d+e x^2}} - \frac{2 e x (a+b \operatorname{ArcSec}[c x])}{d^2 \sqrt{d+e x^2}} -$$

$$\frac{b c^2 x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}} + \frac{b (c^2 d+2 e) x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}$$

Result (type 4, 212 leaves):

$$\frac{b c \sqrt{1-\frac{1}{c^2 x^2}} x (d+e x^2) - a (d+2 e x^2) - b (d+2 e x^2) \operatorname{ArcSec}[c x]}{d^2 x \sqrt{d+e x^2}} -$$

$$\left( \frac{i b c \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{1+\frac{e x^2}{d}} \left( c^2 d \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[\sqrt{-c^2} x\right], -\frac{e}{c^2 d}\right] - (c^2 d+2 e) \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[\sqrt{-c^2} x\right], -\frac{e}{c^2 d}\right] \right)}{\left( \sqrt{-c^2} d^2 \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \right)} \right) /$$

**Problem 150: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSec}[c x]}{x^4 (d+e x^2)^{3/2}} dx$$

Optimal (type 4, 701 leaves, 25 steps):

$$\begin{aligned}
& \frac{2bc(c^2d - e)\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^3\sqrt{c^2x^2}} - \frac{4bce\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{3d^3\sqrt{c^2x^2}} + \frac{bc\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{9d^2x^2\sqrt{c^2x^2}} - \frac{a+b\text{ArcSec}[cx]}{3dx^3\sqrt{d+ex^2}} + \\
& \frac{4e(a+b\text{ArcSec}[cx])}{3d^2x\sqrt{d+ex^2}} + \frac{8e^2x(a+b\text{ArcSec}[cx])}{3d^3\sqrt{d+ex^2}} - \frac{2bc^2(c^2d - e)x\sqrt{1-c^2x^2}\sqrt{d+ex^2}\text{EllipticE}[\text{ArcSin}[cx], -\frac{e}{c^2d}]}{9d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \\
& \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{d+ex^2}\text{EllipticE}[\text{ArcSin}[cx], -\frac{e}{c^2d}]}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{1+\frac{ex^2}{d}}} + \frac{bc^2(2c^2d - e)x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\text{EllipticF}[\text{ArcSin}[cx], -\frac{e}{c^2d}]}{9d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} - \\
& \frac{4bc^2ex\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\text{EllipticF}[\text{ArcSin}[cx], -\frac{e}{c^2d}]}{3d^2\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}} - \frac{8be^2x\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}\text{EllipticF}[\text{ArcSin}[cx], -\frac{e}{c^2d}]}{3d^3\sqrt{c^2x^2}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}
\end{aligned}$$

Result (type 4, 292 leaves):

$$\begin{aligned}
& \frac{1}{9d^3x^3\sqrt{d+ex^2}} \\
& \left( -3a(d^2 - 4dex^2 - 8e^2x^4) + bc\sqrt{1 - \frac{1}{c^2x^2}}x(-14e^2x^4 + dex^2(-13 + 2c^2x^2) + d^2(1 + 2c^2x^2)) - 3b(d^2 - 4dex^2 - 8e^2x^4)\text{ArcSec}[cx] \right) - \\
& \left( ibc\sqrt{1 - \frac{1}{c^2x^2}}x\sqrt{1 + \frac{ex^2}{d}}\left(2c^2d(c^2d - 7e)\text{EllipticE}[i\text{ArcSinh}[\sqrt{-c^2}x], -\frac{e}{c^2d}] + \right. \right. \\
& \left. \left. (-2c^4d^2 + 13c^2de + 24e^2)\text{EllipticF}[i\text{ArcSinh}[\sqrt{-c^2}x], -\frac{e}{c^2d}]\right) \right) / \left( 9\sqrt{-c^2}d^3\sqrt{1-c^2x^2}\sqrt{d+ex^2} \right)
\end{aligned}$$

**Problem 151: Result unnecessarily involves higher level functions.**

$$\int \frac{x^5(a+b\text{ArcSec}[cx])}{(d+ex^2)^{5/2}} dx$$

Optimal (type 3, 244 leaves, 10 steps):

$$\begin{aligned}
& - \frac{b c d x \sqrt{-1+c^2 x^2}}{3 e^2 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{d+e x^2}} - \frac{d^2 (a+b \operatorname{ArcSec}[c x])}{3 e^3 (d+e x^2)^{3/2}} + \frac{2 d (a+b \operatorname{ArcSec}[c x])}{e^3 \sqrt{d+e x^2}} + \\
& \frac{\sqrt{d+e x^2} (a+b \operatorname{ArcSec}[c x])}{e^3} + \frac{8 b c \sqrt{d} x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{3 e^3 \sqrt{c^2 x^2}} - \frac{b x \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1+c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{e^{5/2} \sqrt{c^2 x^2}}
\end{aligned}$$

Result (type 6, 417 leaves):

$$\begin{aligned}
& \left( 2 b c d \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \right. \\
& \left( \left( 8 c^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) / \left( 4 c^2 e x^2 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] - c^2 d \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + \right. \right. \\
& \left. \left. e \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \right) - \left( 3 \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] \right) / \right. \\
& \left. \left( 4 d \operatorname{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left( -e \operatorname{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \operatorname{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, c^2 x^2, -\frac{e x^2}{d}\right] \right) \right) \right) / \\
& \left( 3 e^2 (-1+c^2 x^2) \sqrt{d+e x^2} \right) + \left( -b c d e \sqrt{1 - \frac{1}{c^2 x^2}} x (d+e x^2) + a (c^2 d+e) (8 d^2+12 d e x^2+3 e^2 x^4) + \right. \\
& \left. b (c^2 d+e) (8 d^2+12 d e x^2+3 e^2 x^4) \operatorname{ArcSec}[c x] \right) / \left( 3 e^3 (c^2 d+e) (d+e x^2)^{3/2} \right)
\end{aligned}$$

**Problem 152: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3 (a+b \operatorname{ArcSec}[c x])}{(d+e x^2)^{5/2}} dx$$

Optimal (type 3, 163 leaves, 7 steps):

$$\frac{b c x \sqrt{-1+c^2 x^2}}{3 e (c^2 d+e) \sqrt{c^2 x^2} \sqrt{d+e x^2}} + \frac{d (a+b \operatorname{ArcSec}[c x])}{3 e^2 (d+e x^2)^{3/2}} - \frac{a+b \operatorname{ArcSec}[c x]}{e^2 \sqrt{d+e x^2}} - \frac{2 b c x \operatorname{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1+c^2 x^2}}\right]}{3 \sqrt{d} e^2 \sqrt{c^2 x^2}}$$

Result (type 6, 269 leaves):

$$\begin{aligned}
& - \left( \left( 4 b c^3 \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) / \left( 3 e (-1 + c^2 x^2) \sqrt{d + e x^2} \right. \right. \\
& \quad \left. \left. \left( 4 c^2 e x^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] - c^2 d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] + e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) \right) \right) + \\
& \frac{b c e \sqrt{1 - \frac{1}{c^2 x^2}} x (d + e x^2) - a (c^2 d + e) (2 d + 3 e x^2) - b (c^2 d + e) (2 d + 3 e x^2) \operatorname{ArcSec}[c x]}{3 e^2 (c^2 d + e) (d + e x^2)^{3/2}}
\end{aligned}$$

**Problem 153: Result unnecessarily involves higher level functions.**

$$\int \frac{x (a + b \operatorname{ArcSec}[c x])}{(d + e x^2)^{5/2}} dx$$

Optimal (type 3, 138 leaves, 5 steps):

$$-\frac{b c x \sqrt{-1 + c^2 x^2}}{3 d (c^2 d + e) \sqrt{c^2 x^2} \sqrt{d + e x^2}} - \frac{a + b \operatorname{ArcSec}[c x]}{3 e (d + e x^2)^{3/2}} - \frac{b c x \operatorname{ArcTan} \left[ \frac{\sqrt{d + e x^2}}{\sqrt{d} \sqrt{-1 + c^2 x^2}} \right]}{3 d^{3/2} e \sqrt{c^2 x^2}}$$

Result (type 6, 255 leaves):

$$\begin{aligned}
& - \left( \left( 2 b c^3 \sqrt{1 - \frac{1}{c^2 x^2}} x^3 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) / \left( 3 d (-1 + c^2 x^2) \sqrt{d + e x^2} \right. \right. \\
& \quad \left. \left. \left( 4 c^2 e x^2 \operatorname{AppellF1} \left[ 1, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] - c^2 d \operatorname{AppellF1} \left[ 2, \frac{1}{2}, \frac{3}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] + e \operatorname{AppellF1} \left[ 2, \frac{3}{2}, \frac{1}{2}, 3, \frac{1}{c^2 x^2}, -\frac{d}{e x^2} \right] \right) \right) \right) + \\
& \frac{-a d (c^2 d + e) - b c e \sqrt{1 - \frac{1}{c^2 x^2}} x (d + e x^2) - b d (c^2 d + e) \operatorname{ArcSec}[c x]}{3 d e (c^2 d + e) (d + e x^2)^{3/2}}
\end{aligned}$$

**Problem 159: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSec}[c x]}{(d + e x^2)^{5/2}} dx$$

Optimal (type 4, 296 leaves, 10 steps):

$$\frac{b c e x^2 \sqrt{-1+c^2 x^2}}{3 d^2 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{d+e x^2}} + \frac{x (a+b \operatorname{ArcSec}[c x])}{3 d (d+e x^2)^{3/2}} + \frac{2 x (a+b \operatorname{ArcSec}[c x])}{3 d^2 \sqrt{d+e x^2}} -$$

$$\frac{b c^2 x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 d^2 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{1+\frac{e x^2}{d}}} - \frac{2 b x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}$$

Result (type 4, 248 leaves):

$$\frac{x \left( b c e \sqrt{1-\frac{1}{c^2 x^2}} x (d+e x^2) + a (c^2 d+e) (3 d+2 e x^2) + b (c^2 d+e) (3 d+2 e x^2) \operatorname{ArcSec}[c x] \right)}{3 d^2 (c^2 d+e) (d+e x^2)^{3/2}} -$$

$$\left( i b c \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{1+\frac{e x^2}{d}} \left( c^2 d \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] + 2 (c^2 d+e) \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] \right) \right) /$$

$$\left( 3 \sqrt{-c^2} d^2 (c^2 d+e) \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \right)$$

**Problem 160: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSec}[c x]}{x^2 (d+e x^2)^{5/2}} dx$$

Optimal (type 4, 631 leaves, 26 steps):

$$\begin{aligned}
& - \frac{b c e \sqrt{-1+c^2 x^2}}{d^2 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{d+e x^2}} - \frac{4 b c e^2 x^2 \sqrt{-1+c^2 x^2}}{3 d^3 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{d+e x^2}} + \\
& \frac{b c (c^2 d+2 e) \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{d^3 (c^2 d+e) \sqrt{c^2 x^2}} - \frac{a+b \operatorname{ArcSec}[c x]}{d x (d+e x^2)^{3/2}} - \frac{4 e x (a+b \operatorname{ArcSec}[c x])}{3 d^2 (d+e x^2)^{3/2}} - \frac{8 e x (a+b \operatorname{ArcSec}[c x])}{3 d^3 \sqrt{d+e x^2}} + \\
& \frac{4 b c^2 e x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 d^3 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2}} - \frac{b c^2 (c^2 d+2 e) x \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{d^3 (c^2 d+e) \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2}} \sqrt{1+\frac{e x^2}{d}} \\
& \frac{b c^2 x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{d^2 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}} + \frac{8 b e x \sqrt{1-c^2 x^2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 d^3 \sqrt{c^2 x^2} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}
\end{aligned}$$

Result (type 4, 323 leaves):

$$\begin{aligned}
& \left( -a (c^2 d+e) (3 d^2+12 d e x^2+8 e^2 x^4) + b c \sqrt{1-\frac{1}{c^2 x^2}} x (d+e x^2) (3 c^2 d (d+e x^2)+e (3 d+2 e x^2)) - \right. \\
& \left. b (c^2 d+e) (3 d^2+12 d e x^2+8 e^2 x^4) \operatorname{ArcSec}[c x] \right) / \left( 3 d^3 (c^2 d+e) x (d+e x^2)^{3/2} - \right. \\
& \left. \left( i b c \sqrt{1-\frac{1}{c^2 x^2}} x \sqrt{1+\frac{e x^2}{d}} (c^2 d (3 c^2 d+2 e) \operatorname{EllipticE}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] - \right. \right. \\
& \left. \left. (3 c^4 d^2+11 c^2 d e+8 e^2) \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{-c^2} x], -\frac{e}{c^2 d}] \right) \right) / \left( 3 \sqrt{-c^2} d^3 (c^2 d+e) \sqrt{1-c^2 x^2} \sqrt{d+e x^2} \right)
\end{aligned}$$

## Test results for the 50 problems in "5.5.2 Inverse secant functions.m"

Problem 14: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSec}\left[\frac{a}{x}\right]}{x^2} dx$$

Optimal (type 3, 31 leaves, 5 steps):

$$-\frac{\text{ArcCos}\left[\frac{x}{a}\right]}{x} + \frac{\text{ArcTanh}\left[\sqrt{1 - \frac{x^2}{a^2}}\right]}{a}$$

Result (type 3, 93 leaves):

$$-\frac{\text{ArcSec}\left[\frac{a}{x}\right]}{x} + \frac{\sqrt{-1 + \frac{a^2}{x^2}} \times \left( -\text{Log}\left[1 - \frac{a}{\sqrt{-1 + \frac{a^2}{x^2}} x}\right] + \text{Log}\left[1 + \frac{a}{\sqrt{-1 + \frac{a^2}{x^2}} x}\right] \right)}{2 a^2 \sqrt{1 - \frac{x^2}{a^2}}}$$

**Problem 17: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{ArcSec}[a x^n]}{x} dx$$

Optimal (type 4, 69 leaves, 7 steps):

$$\frac{i \text{ArcSec}[a x^n]^2}{2 n} - \frac{\text{ArcSec}[a x^n] \text{Log}\left[1 + e^{2 i \text{ArcSec}[a x^n]}\right]}{n} + \frac{i \text{PolyLog}\left[2, -e^{2 i \text{ArcSec}[a x^n]}\right]}{2 n}$$

Result (type 5, 60 leaves):

$$\frac{x^{-n} \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, \frac{x^{-2n}}{a^2}\right]}{a n} + \left( \text{ArcSec}[a x^n] + \text{ArcSin}\left[\frac{x^{-n}}{a}\right] \right) \text{Log}[x]$$

**Problem 22: Result more than twice size of optimal antiderivative.**

$$\int \text{ArcSec}[a + b x] dx$$

Optimal (type 3, 37 leaves, 5 steps):

$$\frac{(a + b x) \text{ArcSec}[a + b x]}{b} - \frac{\text{ArcTanh}\left[\sqrt{1 - \frac{1}{(a + b x)^2}}\right]}{b}$$

Result (type 3, 121 leaves):

$$x \text{ArcSec}[a + b x] - \frac{(a + b x) \sqrt{\frac{-1 + a^2 + 2 a b x + b^2 x^2}{(a + b x)^2}} \left( a \text{ArcTan}\left[\frac{1}{\sqrt{-1 + a^2 + 2 a b x + b^2 x^2}}\right] + \text{Log}\left[a + b x + \sqrt{-1 + a^2 + 2 a b x + b^2 x^2}\right] \right)}{b \sqrt{-1 + a^2 + 2 a b x + b^2 x^2}}$$

**Problem 24:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcSec}[a + b x]}{x^2} dx$$

Optimal (type 3, 70 leaves, 5 steps):

$$-\frac{b \text{ArcSec}[a + b x]}{a} - \frac{\text{ArcSec}[a + b x]}{x} + \frac{2 b \text{ArcTan}\left[\frac{\sqrt{1+a} \tan\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]}{\sqrt{1-a}}\right]}{a \sqrt{1-a^2}}$$

Result (type 3, 112 leaves):

$$-\frac{\text{ArcSec}[a + b x]}{x} + \frac{b \left( \text{ArcSin}\left[\frac{1}{a + b x}\right] - \frac{i \text{Log}\left[\frac{2 \left( \frac{i a (-1 + a^2 + a b x)}{\sqrt{1-a^2}} + a (a + b x) \sqrt{\frac{-1 + a^2 + 2 a b x + b^2 x^2}{(a + b x)^2}}\right)}{b x \sqrt{1-a^2}}\right]}{a} \right)}{a}$$

**Problem 25:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcSec}[a + b x]}{x^3} dx$$

Optimal (type 3, 125 leaves, 7 steps):

$$\frac{b (a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}}}{2 a (1 - a^2) x} + \frac{b^2 \text{ArcSec}[a + b x]}{2 a^2} - \frac{\text{ArcSec}[a + b x]}{2 x^2} - \frac{(1 - 2 a^2) b^2 \text{ArcTan}\left[\frac{\sqrt{1+a} \tan\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]}{\sqrt{1-a}}\right]}{a^2 (1 - a^2)^{3/2}}$$

Result (type 3, 198 leaves):

$$-\frac{1}{2 x^2} \left( \frac{b x (a + b x) \sqrt{\frac{-1 + a^2 + 2 a b x + b^2 x^2}{(a + b x)^2}}}{a (-1 + a^2)} + \text{ArcSec}[a + b x] + \frac{b^2 x^2 \text{ArcSin}\left[\frac{1}{a + b x}\right]}{a^2} + \frac{i (-1 + 2 a^2) b^2 x^2 \text{Log}\left[\frac{4 (-1 + a) a^2 (1 + a) \left( -\frac{i (-1 + a^2 + a b x)}{\sqrt{1-a^2}} - (a + b x) \sqrt{\frac{-1 + a^2 + 2 a b x + b^2 x^2}{(a + b x)^2}} \right)}{(-1 + 2 a^2) b^2 x}\right]}{a^2 (1 - a^2)^{3/2}} \right)$$

**Problem 26:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{ArcSec}[a + b x]}{x^4} dx$$



Optimal (type 3, 181 leaves, 8 steps):

$$\frac{b(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a(1-a^2)x^2} - \frac{(2-5a^2)b^2(a+bx)\sqrt{1-\frac{1}{(a+bx)^2}}}{6a^2(1-a^2)^2x}$$

$$\frac{b^3 \operatorname{ArcSec}[a+bx]}{3a^3} - \frac{\operatorname{ArcSec}[a+bx]}{3x^3} + \frac{(2-5a^2+6a^4)b^3 \operatorname{ArcTan}\left[\frac{\sqrt{1+a}\operatorname{Tan}\left[\frac{1}{2}\operatorname{ArcSec}[a+bx]\right]}{\sqrt{1-a}}\right]}{3a^3(1-a^2)^{5/2}}$$

Result (type 3, 241 leaves):

$$\frac{1}{6} \left( \frac{b\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}(a^4+abx-4a^3bx+2b^2x^2-a^2(1+5b^2x^2))}{a^2(-1+a^2)^2x^2} - \frac{2 \operatorname{ArcSec}[a+bx]}{x^3} + \frac{2b^3 \operatorname{ArcSin}\left[\frac{1}{a+bx}\right]}{a^3} - \frac{i(2-5a^2+6a^4)b^3 \operatorname{Log}\left[\frac{12a^3(-1+a^2)^2\left(\frac{i(-1+a^2+abx)}{\sqrt{1-a^2}}+(a+bx)\sqrt{\frac{-1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right)}{(2-5a^2+6a^4)b^3x}\right]}{a^3(1-a^2)^{5/2}} \right)$$

**Problem 27: Result more than twice size of optimal antiderivative.**

$$\int x^3 \operatorname{ArcSec}[a+bx]^2 dx$$

Optimal (type 4, 381 leaves, 20 steps):

$$\begin{aligned}
& -\frac{a x}{b^3} + \frac{(a+b x)^2}{12 b^4} - \frac{(a+b x) \sqrt{1-\frac{1}{(a+b x)^2}} \operatorname{ArcSec}[a+b x]}{3 b^4} - \frac{3 a^2 (a+b x) \sqrt{1-\frac{1}{(a+b x)^2}} \operatorname{ArcSec}[a+b x]}{b^4} + \\
& \frac{a (a+b x)^2 \sqrt{1-\frac{1}{(a+b x)^2}} \operatorname{ArcSec}[a+b x]}{b^4} - \frac{(a+b x)^3 \sqrt{1-\frac{1}{(a+b x)^2}} \operatorname{ArcSec}[a+b x]}{6 b^4} - \frac{a^4 \operatorname{ArcSec}[a+b x]^2}{4 b^4} + \frac{1}{4} x^4 \operatorname{ArcSec}[a+b x]^2 - \\
& \frac{2 i a \operatorname{ArcSec}[a+b x] \operatorname{ArcTan}\left[e^{i \operatorname{ArcSec}[a+b x]}\right]}{b^4} - \frac{4 i a^3 \operatorname{ArcSec}[a+b x] \operatorname{ArcTan}\left[e^{i \operatorname{ArcSec}[a+b x]}\right]}{b^4} + \frac{\operatorname{Log}[a+b x]}{3 b^4} + \frac{3 a^2 \operatorname{Log}[a+b x]}{b^4} + \\
& \frac{i a \operatorname{PolyLog}\left[2, -i e^{i \operatorname{ArcSec}[a+b x]}\right]}{b^4} + \frac{2 i a^3 \operatorname{PolyLog}\left[2, -i e^{i \operatorname{ArcSec}[a+b x]}\right]}{b^4} - \frac{i a \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcSec}[a+b x]}\right]}{b^4} - \frac{2 i a^3 \operatorname{PolyLog}\left[2, i e^{i \operatorname{ArcSec}[a+b x]}\right]}{b^4}
\end{aligned}$$

Result (type 4, 1141 leaves):

$$\begin{aligned}
& \frac{1}{b^4} \left( \frac{a b^3 x^3 (2 + \operatorname{ArcSec}[a+b x]^2 + 2 a^2 \operatorname{ArcSec}[a+b x]^2)}{2 (a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3} - \frac{\left(-\frac{1}{3} - 3 a^2\right) b^3 x^3 \operatorname{Log}\left[\frac{1}{a+b x}\right]}{(a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3} + \right. \\
& \frac{1}{(a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3} \left( -a - 2 a^3 \right) b^3 x^3 \left( \left( \frac{\pi}{2} - \operatorname{ArcSec}[a+b x] \right) \left( \operatorname{Log}\left[1 - e^{i \left(\frac{\pi}{2} - \operatorname{ArcSec}[a+b x]\right)}\right] - \operatorname{Log}\left[1 + e^{i \left(\frac{\pi}{2} - \operatorname{ArcSec}[a+b x]\right)}\right] \right) - \right. \\
& \left. \left. \frac{1}{2} \pi \operatorname{Log}\left[\operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - \operatorname{ArcSec}[a+b x]\right)\right]\right] + i \left( \operatorname{PolyLog}\left[2, -e^{i \left(\frac{\pi}{2} - \operatorname{ArcSec}[a+b x]\right)}\right] - \operatorname{PolyLog}\left[2, e^{i \left(\frac{\pi}{2} - \operatorname{ArcSec}[a+b x]\right)}\right] \right) \right) \right) - \\
& \frac{b^3 x^3 \operatorname{ArcSec}[a+b x]^2}{16 (a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3 \left( \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] \right)^4} + \\
& \left( b^3 x^3 \left( -2 + 2 \operatorname{ArcSec}[a+b x] - 24 a \operatorname{ArcSec}[a+b x] - 3 \operatorname{ArcSec}[a+b x]^2 + 12 a \operatorname{ArcSec}[a+b x]^2 - 36 a^2 \operatorname{ArcSec}[a+b x]^2 \right) \right) / \\
& \left( 48 (a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3 \left( \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] - \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] \right)^2 \right) - \\
& \frac{b^3 x^3 \operatorname{ArcSec}[a+b x]^2}{16 (a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3 \left( \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] \right)^4} + \\
& \left( b^3 x^3 \left( -2 - 2 \operatorname{ArcSec}[a+b x] + 24 a \operatorname{ArcSec}[a+b x] - 3 \operatorname{ArcSec}[a+b x]^2 + 12 a \operatorname{ArcSec}[a+b x]^2 - 36 a^2 \operatorname{ArcSec}[a+b x]^2 \right) \right) / \\
& \left( 48 (a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3 \left( \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] \right)^2 \right) + \\
& \frac{b^3 x^3 \left( \operatorname{ArcSec}[a+b x] \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] - 6 a \operatorname{ArcSec}[a+b x]^2 \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] \right)}{12 (a+b x)^3 \left(-1 + \frac{a}{a+b x}\right)^3 \left( \operatorname{Cos}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] + \operatorname{Sin}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right] \right)^3} +
\end{aligned}$$

$$\begin{aligned}
& \frac{b^3 x^3 \left( \text{ArcSec}[a + b x] \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + 6 a \text{ArcSec}[a + b x]^2 \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] \right)}{12 (a + b x)^3 \left(-1 + \frac{a}{a + b x}\right)^3 \left(\cos\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] - \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]\right)^3} + \\
& \left( b^3 x^3 \left( -6 a \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + 2 \text{ArcSec}[a + b x] \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + 18 a^2 \text{ArcSec}[a + b x] \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] - \right. \right. \\
& \quad \left. \left. 3 a \text{ArcSec}[a + b x]^2 \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] - 6 a^3 \text{ArcSec}[a + b x]^2 \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] \right) \right) / \\
& \left( 6 (a + b x)^3 \left(-1 + \frac{a}{a + b x}\right)^3 \left(\cos\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]\right) \right) + \\
& \left( b^3 x^3 \left( 6 a \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + 2 \text{ArcSec}[a + b x] \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + 18 a^2 \text{ArcSec}[a + b x] \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + \right. \right. \\
& \quad \left. \left. 3 a \text{ArcSec}[a + b x]^2 \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] + 6 a^3 \text{ArcSec}[a + b x]^2 \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] \right) \right) / \\
& \left( 6 (a + b x)^3 \left(-1 + \frac{a}{a + b x}\right)^3 \left(\cos\left[\frac{1}{2} \text{ArcSec}[a + b x]\right] - \sin\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]\right) \right) \Big)
\end{aligned}$$

**Problem 31: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcSec}[a + b x]^2}{x} dx$$

Optimal (type 4, 310 leaves, 17 steps):

$$\begin{aligned}
& \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] + \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \text{ArcSec}[a + b x]^2 \text{Log}\left[1 + e^{2 i \text{ArcSec}[a + b x]}\right] - \\
& 2 i \text{ArcSec}[a + b x] \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] - 2 i \text{ArcSec}[a + b x] \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] + \\
& i \text{ArcSec}[a + b x] \text{PolyLog}\left[2, -e^{2 i \text{ArcSec}[a + b x]}\right] + 2 \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] + 2 \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \frac{1}{2} \text{PolyLog}\left[3, -e^{2 i \text{ArcSec}[a + b x]}\right]
\end{aligned}$$

Result (type 4, 813 leaves):

$$\begin{aligned}
& \text{ArcSec}[a + b x]^2 \text{Log}\left[1 + \frac{a e^{i \text{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] + \text{ArcSec}[a + b x]^2 \text{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2}) e^{i \text{ArcSec}[a + b x]}}{a}\right] - \\
& 4 \text{ArcSec}[a + b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2}) e^{i \text{ArcSec}[a + b x]}}{a}\right] + \\
& \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] + \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2}) e^{i \text{ArcSec}[a + b x]}}{a}\right] + \\
& 4 \text{ArcSec}[a + b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2}) e^{i \text{ArcSec}[a + b x]}}{a}\right] - 2 \text{ArcSec}[a + b x]^2 \text{Log}\left[1 + e^{2 i \text{ArcSec}[a + b x]}\right] + \\
& \text{ArcSec}[a + b x]^2 \text{Log}\left[\frac{2\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a + b x}\right] - \text{ArcSec}[a + b x]^2 \text{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] + \\
& 4 \text{ArcSec}[a + b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] - \\
& \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] - \\
& 4 \text{ArcSec}[a + b x] \text{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] - 2 i \text{ArcSec}[a + b x] \text{PolyLog}\left[2, -\frac{a e^{i \text{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] - \\
& 2 i \text{ArcSec}[a + b x] \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] + i \text{ArcSec}[a + b x] \text{PolyLog}\left[2, -e^{2 i \text{ArcSec}[a + b x]}\right] + \\
& 2 \text{PolyLog}\left[3, -\frac{a e^{i \text{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] + 2 \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \frac{1}{2} \text{PolyLog}\left[3, -e^{2 i \text{ArcSec}[a + b x]}\right]
\end{aligned}$$

**Problem 32: Result more than twice size of optimal antiderivative.**

$$\int \frac{\text{ArcSec}[a + b x]^2}{x^2} dx$$

Optimal (type 4, 244 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{b \operatorname{ArcSec}[a + b x]^2}{a} - \frac{\operatorname{ArcSec}[a + b x]^2}{x} - \frac{2 i b \operatorname{ArcSec}[a + b x] \operatorname{Log}\left[1 - \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} + \\
 & \frac{2 i b \operatorname{ArcSec}[a + b x] \operatorname{Log}\left[1 - \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} - \frac{2 b \operatorname{PolyLog}\left[2, \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} + \frac{2 b \operatorname{PolyLog}\left[2, \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}}
 \end{aligned}$$

Result (type 4, 686 leaves):

$$\begin{aligned}
& -\frac{1}{a} \left( \frac{(a+bx) \operatorname{ArcSec}[a+bx]^2}{x} + \right. \\
& \frac{1}{\sqrt{-1+a^2}} 2b \left( 2 \operatorname{ArcSec}[a+bx] \operatorname{ArcTanh} \left[ \frac{(-1+a) \operatorname{Cot} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] - 2 \operatorname{ArcCos} \left[ \frac{1}{a} \right] \operatorname{ArcTanh} \left[ \frac{(1+a) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] + \right. \\
& \left. \left( \operatorname{ArcCos} \left[ \frac{1}{a} \right] - 2i \operatorname{ArcTanh} \left[ \frac{(-1+a) \operatorname{Cot} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] + 2i \operatorname{ArcTanh} \left[ \frac{(1+a) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] \right) \right. \\
& \left. \operatorname{Log} \left[ \frac{\sqrt{-1+a^2} e^{-\frac{1}{2}i \operatorname{ArcSec}[a+bx]}}{\sqrt{2} \sqrt{a} \sqrt{-\frac{bx}{a+bx}}} \right] + \left( \operatorname{ArcCos} \left[ \frac{1}{a} \right] + \right. \right. \\
& \left. \left. 2i \left( \operatorname{ArcTanh} \left[ \frac{(-1+a) \operatorname{Cot} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] - \operatorname{ArcTanh} \left[ \frac{(1+a) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] \right) \right) \operatorname{Log} \left[ \frac{\sqrt{-1+a^2} e^{\frac{1}{2}i \operatorname{ArcSec}[a+bx]}}{\sqrt{2} \sqrt{a} \sqrt{-\frac{bx}{a+bx}}} \right] - \right. \\
& \left. \left( \operatorname{ArcCos} \left[ \frac{1}{a} \right] - 2i \operatorname{ArcTanh} \left[ \frac{(1+a) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] \right) \operatorname{Log} \left[ \frac{(-1+a) (i+i a + \sqrt{-1+a^2}) (-i + \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])}{a (-1+a + \sqrt{-1+a^2} \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])} \right] - \right. \\
& \left. \left( \operatorname{ArcCos} \left[ \frac{1}{a} \right] + 2i \operatorname{ArcTanh} \left[ \frac{(1+a) \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right]}{\sqrt{-1+a^2}} \right] \right) \operatorname{Log} \left[ \frac{(-1+a) (-i-i a + \sqrt{-1+a^2}) (i + \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])}{a (-1+a + \sqrt{-1+a^2} \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])} \right] + \right. \\
& \left. i \left( -\operatorname{PolyLog} \left[ 2, \frac{(1-i\sqrt{-1+a^2}) (1-a + \sqrt{-1+a^2} \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])}{a (-1+a + \sqrt{-1+a^2} \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])} \right] + \right. \right. \\
& \left. \left. \operatorname{PolyLog} \left[ 2, \frac{(1+i\sqrt{-1+a^2}) (1-a + \sqrt{-1+a^2} \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])}{a (-1+a + \sqrt{-1+a^2} \operatorname{Tan} \left[ \frac{1}{2} \operatorname{ArcSec}[a+bx] \right])} \right] \right) \right) \right) \right)
\end{aligned}$$

### Problem 33: Unable to integrate problem.

$$\int x^2 \operatorname{ArcSec}[a + b x]^3 dx$$

Optimal (type 4, 494 leaves, 25 steps):

$$\begin{aligned} & \frac{(a + b x) \operatorname{ArcSec}[a + b x]}{b^3} - \frac{3 i a \operatorname{ArcSec}[a + b x]^2}{b^3} + \frac{3 a (a + b x) \sqrt{1 - \frac{1}{(a + b x)^2}} \operatorname{ArcSec}[a + b x]^2}{b^3} - \\ & \frac{(a + b x)^2 \sqrt{1 - \frac{1}{(a + b x)^2}} \operatorname{ArcSec}[a + b x]^2}{2 b^3} + \frac{a^3 \operatorname{ArcSec}[a + b x]^3}{3 b^3} + \frac{1}{3} x^3 \operatorname{ArcSec}[a + b x]^3 + \frac{i \operatorname{ArcSec}[a + b x]^2 \operatorname{ArcTan}[e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} + \\ & \frac{6 i a^2 \operatorname{ArcSec}[a + b x]^2 \operatorname{ArcTan}[e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} - \frac{\operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{(a + b x)^2}}\right]}{b^3} + \frac{6 a \operatorname{ArcSec}[a + b x] \operatorname{Log}[1 + e^{2 i \operatorname{ArcSec}[a + b x]}]}{b^3} - \\ & \frac{i \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} - \frac{6 i a^2 \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}[2, -i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} + \\ & \frac{i \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} + \frac{6 i a^2 \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}[2, i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} - \frac{3 i a \operatorname{PolyLog}[2, -e^{2 i \operatorname{ArcSec}[a + b x]}]}{b^3} + \\ & \frac{\operatorname{PolyLog}[3, -i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} + \frac{6 a^2 \operatorname{PolyLog}[3, -i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} - \frac{\operatorname{PolyLog}[3, i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} - \frac{6 a^2 \operatorname{PolyLog}[3, i e^{i \operatorname{ArcSec}[a + b x]}]}{b^3} \end{aligned}$$

Result (type 8, 14 leaves):

$$\int x^2 \operatorname{ArcSec}[a + b x]^3 dx$$

### Problem 36: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcSec}[a + b x]^3}{x} dx$$

Optimal (type 4, 430 leaves, 20 steps):

$$\begin{aligned}
& \text{ArcSec}[a + b x]^3 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] + \text{ArcSec}[a + b x]^3 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \text{ArcSec}[a + b x]^3 \text{Log}\left[1 + e^{2 i \text{ArcSec}[a + b x]}\right] - \\
& 3 i \text{ArcSec}[a + b x]^2 \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] - 3 i \text{ArcSec}[a + b x]^2 \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] + \\
& \frac{3}{2} i \text{ArcSec}[a + b x]^2 \text{PolyLog}\left[2, -e^{2 i \text{ArcSec}[a + b x]}\right] + 6 \text{ArcSec}[a + b x] \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] + \\
& 6 \text{ArcSec}[a + b x] \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \frac{3}{2} \text{ArcSec}[a + b x] \text{PolyLog}\left[3, -e^{2 i \text{ArcSec}[a + b x]}\right] + \\
& 6 i \text{PolyLog}\left[4, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right] + 6 i \text{PolyLog}\left[4, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \frac{3}{4} i \text{PolyLog}\left[4, -e^{2 i \text{ArcSec}[a + b x]}\right]
\end{aligned}$$

Result (type 4, 1058 leaves):



$$\begin{aligned}
& 2 \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 + \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] + \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2}) e^{i \operatorname{ArcSec}[a + b x]}}{a}\right] - \\
& 6 \operatorname{ArcSec}[a + b x]^2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2}) e^{i \operatorname{ArcSec}[a + b x]}}{a}\right] + 2 \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 - \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] + \\
& \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2}) e^{i \operatorname{ArcSec}[a + b x]}}{a}\right] + 6 \operatorname{ArcSec}[a + b x]^2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2}) e^{i \operatorname{ArcSec}[a + b x]}}{a}\right] - \\
& 3 \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 + e^{2 i \operatorname{ArcSec}[a + b x]}\right] + 2 \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[\frac{2\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a + b x}\right] - \\
& \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 + \frac{a\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{-1 + \sqrt{1 - a^2}}\right] - \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] + \\
& 6 \operatorname{ArcSec}[a + b x]^2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(-1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] - \\
& \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 - \frac{a\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{1 + \sqrt{1 - a^2}}\right] - \operatorname{ArcSec}[a + b x]^3 \operatorname{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] - \\
& 6 \operatorname{ArcSec}[a + b x]^2 \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-1+a}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(1 + \sqrt{1 - a^2})\left(\frac{1}{a + b x} + i \sqrt{1 - \frac{1}{(a + b x)^2}}\right)}{a}\right] - 3 i \operatorname{ArcSec}[a + b x]^2 \operatorname{PolyLog}\left[2, -\frac{a e^{i \operatorname{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] - \\
& 3 i \operatorname{ArcSec}[a + b x]^2 \operatorname{PolyLog}\left[2, \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] + \frac{3}{2} i \operatorname{ArcSec}[a + b x]^2 \operatorname{PolyLog}\left[2, -e^{2 i \operatorname{ArcSec}[a + b x]}\right] + \\
& 6 \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}\left[3, -\frac{a e^{i \operatorname{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] + 6 \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}\left[3, \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \frac{3}{2} \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}\left[3, -e^{2 i \operatorname{ArcSec}[a + b x]}\right] + \\
& 6 i \operatorname{PolyLog}\left[4, -\frac{a e^{i \operatorname{ArcSec}[a + b x]}}{-1 + \sqrt{1 - a^2}}\right] + 6 i \operatorname{PolyLog}\left[4, \frac{a e^{i \operatorname{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right] - \frac{3}{4} i \operatorname{PolyLog}\left[4, -e^{2 i \operatorname{ArcSec}[a + b x]}\right]
\end{aligned}$$

### Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcSec}[a + b x]^3}{x^2} dx$$

Optimal (type 4, 362 leaves, 14 steps):

$$\begin{aligned} & -\frac{b \text{ArcSec}[a + b x]^3}{a} - \frac{\text{ArcSec}[a + b x]^3}{x} - \frac{3 i b \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} + \\ & \frac{3 i b \text{ArcSec}[a + b x]^2 \text{Log}\left[1 - \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} - \frac{6 b \text{ArcSec}[a + b x] \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} + \\ & \frac{6 b \text{ArcSec}[a + b x] \text{PolyLog}\left[2, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} - \frac{6 i b \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 - \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} + \frac{6 i b \text{PolyLog}\left[3, \frac{a e^{i \text{ArcSec}[a + b x]}}{1 + \sqrt{1 - a^2}}\right]}{a \sqrt{1 - a^2}} \end{aligned}$$

Result (type 4, 1664 leaves):

$$\begin{aligned} & -\frac{1}{a \sqrt{-1 + a^2} x} \left( a \sqrt{-1 + a^2} \text{ArcSec}[a + b x]^3 + \sqrt{-1 + a^2} b x \text{ArcSec}[a + b x]^3 + \right. \\ & \left. 6 b x \text{ArcCos}\left[-\frac{1}{a}\right] \text{ArcSec}[a + b x] \text{Log}\left[\frac{\sqrt{-1 + a^2} e^{-\text{ArcTanh}\left[\frac{(1+a) \text{Tan}\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]}{\sqrt{-1 + a^2}}\right]}}{\sqrt{2} \sqrt{a} \sqrt{1 + a \text{Cosh}\left[2 \text{ArcTanh}\left[\frac{(1+a) \text{Tan}\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]}{\sqrt{-1 + a^2}}\right]}}}\right]\right] + \right. \\ & \left. 12 b x \text{ArcSec}[a + b x] \text{ArcTan}\left[\text{Cot}\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]\right] \text{Log}\left[\frac{\sqrt{-1 + a^2} e^{-\text{ArcTanh}\left[\frac{(1+a) \text{Tan}\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]}{\sqrt{-1 + a^2}}\right]}}{\sqrt{2} \sqrt{a} \sqrt{1 + a \text{Cosh}\left[2 \text{ArcTanh}\left[\frac{(1+a) \text{Tan}\left[\frac{1}{2} \text{ArcSec}[a + b x]\right]}{\sqrt{-1 + a^2}}\right]}}}\right]\right] + \right. \end{aligned}$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} e^{-\operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2} \sqrt{a} \sqrt{1+a \operatorname{Cosh}\left[2 \operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right] +$$

$$6 b x \operatorname{ArcCos}\left[-\frac{1}{a}\right] \operatorname{ArcSec}[a + b x] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} e^{\operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2} \sqrt{a} \sqrt{1+a \operatorname{Cosh}\left[2 \operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right] -$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} e^{\operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2} \sqrt{a} \sqrt{1+a \operatorname{Cosh}\left[2 \operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right] -$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} e^{\operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]}}{\sqrt{2} \sqrt{a} \sqrt{1+a \operatorname{Cosh}\left[2 \operatorname{ArcTanh}\left[\frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right]\right]}}\right] -$$

$$6 b x \operatorname{ArcCos}\left[-\frac{1}{a}\right] \operatorname{ArcSec}[a + b x] \operatorname{Log}\left[\frac{(-1+a^2) \sqrt{-\frac{b x}{(-1+a)(1+a+b x)}}}{\sqrt{a} \sqrt{-\frac{(-1+a^2)(a+b x)}{b x}} \left(\sqrt{-1+a^2} + (1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right)}\right] -$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{(-1+a^2) \sqrt{-\frac{b x}{(-1+a)(1+a+b x)}}}{\sqrt{a} \sqrt{-\frac{(-1+a^2)(a+b x)}{b x}} \left(\sqrt{-1+a^2} + (1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right)}\right] -$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{(-1 + a^2) \sqrt{-\frac{b x}{(-1+a)(1+a+b x)}}}{\sqrt{a} \sqrt{-\frac{(-1+a^2)(a+b x)}{b x}} \left(\sqrt{-1+a^2} + (1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right)}\right] -$$

$$6 b x \operatorname{ArcCos}\left[-\frac{1}{a}\right] \operatorname{ArcSec}[a + b x] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} + (1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]}{2 \sqrt{a} \sqrt{-\frac{(-1+a^2)(a+b x)}{b x}} \sqrt{-\frac{b x}{(-1+a)(1+a+b x)}}}\right] +$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Cot}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} + (1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]}{2 \sqrt{a} \sqrt{-\frac{(-1+a^2)(a+b x)}{b x}} \sqrt{-\frac{b x}{(-1+a)(1+a+b x)}}}\right] +$$

$$12 b x \operatorname{ArcSec}[a + b x] \operatorname{ArcTan}\left[\operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]\right] \operatorname{Log}\left[\frac{\sqrt{-1+a^2} + (1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]}{2 \sqrt{a} \sqrt{-\frac{(-1+a^2)(a+b x)}{b x}} \sqrt{-\frac{b x}{(-1+a)(1+a+b x)}}}\right] -$$

$$3 b x \operatorname{ArcSec}[a + b x]^2 \operatorname{Log}\left[-\frac{\left(-1 + a - i \sqrt{-1 + a^2}\right) \left(-1 + \frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right)}{a + i a \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]}\right] +$$

$$3 b x \operatorname{ArcSec}[a + b x]^2 \operatorname{Log}\left[\frac{\left(-1 + a + i \sqrt{-1 + a^2}\right) \left(1 + \frac{(1+a) \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a+b x]\right]}{\sqrt{-1+a^2}}\right)}{a + i a \operatorname{Tan}\left[\frac{1}{2} \operatorname{ArcSec}[a + b x]\right]}\right] + 6 i b x \operatorname{ArcSec}[a + b x]$$

$$\operatorname{PolyLog}\left[2, \frac{\left(1 - i \sqrt{-1 + a^2}\right) \left(\frac{1}{a+b x} - i \sqrt{1 - \frac{1}{(a+b x)^2}}\right)}{a}\right] - 6 i b x \operatorname{ArcSec}[a + b x] \operatorname{PolyLog}\left[2, \frac{\left(1 + i \sqrt{-1 + a^2}\right) \left(\frac{1}{a+b x} - i \sqrt{1 - \frac{1}{(a+b x)^2}}\right)}{a}\right] +$$

$$6 b x \operatorname{PolyLog}\left[3, \frac{\left(1 - i \sqrt{-1 + a^2}\right) \left(\frac{1}{a+b x} - i \sqrt{1 - \frac{1}{(a+b x)^2}}\right)}{a}\right] - 6 b x \operatorname{PolyLog}\left[3, \frac{\left(1 + i \sqrt{-1 + a^2}\right) \left(\frac{1}{a+b x} - i \sqrt{1 - \frac{1}{(a+b x)^2}}\right)}{a}\right] \right]$$

### Problem 38: Result more than twice size of optimal antiderivative.

$$\int x (a + b \operatorname{ArcSec}[c + d x^2]) dx$$

Optimal (type 3, 58 leaves, 7 steps):

$$\frac{a x^2}{2} + \frac{b (c + d x^2) \operatorname{ArcSec}[c + d x^2]}{2 d} - \frac{b \operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{(c + d x^2)^2}}\right]}{2 d}$$

Result (type 3, 154 leaves):

$$\frac{a x^2}{2} + \frac{1}{2} b x^2 \operatorname{ArcSec}[c + d x^2] - \frac{b (c + d x^2) \sqrt{\frac{-1 + c^2 + 2 c d x^2 + d^2 x^4}{(c + d x^2)^2}} \left( c \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1 + c^2 + 2 c d x^2 + d^2 x^4}}\right] + \operatorname{Log}\left[c + d x^2 + \sqrt{-1 + c^2 + 2 c d x^2 + d^2 x^4}\right] \right)}{2 d \sqrt{-1 + c^2 + 2 c d x^2 + d^2 x^4}}$$

### Problem 39: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b \operatorname{ArcSec}[c + d x^3]) dx$$

Optimal (type 3, 58 leaves, 7 steps):

$$\frac{a x^3}{3} + \frac{b (c + d x^3) \operatorname{ArcSec}[c + d x^3]}{3 d} - \frac{b \operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{(c + d x^3)^2}}\right]}{3 d}$$

Result (type 3, 154 leaves):

$$\frac{a x^3}{3} + \frac{1}{3} b x^3 \operatorname{ArcSec}[c + d x^3] - \frac{b (c + d x^3) \sqrt{\frac{-1 + c^2 + 2 c d x^3 + d^2 x^6}{(c + d x^3)^2}} \left( c \operatorname{ArcTan}\left[\frac{1}{\sqrt{-1 + c^2 + 2 c d x^3 + d^2 x^6}}\right] + \operatorname{Log}\left[c + d x^3 + \sqrt{-1 + c^2 + 2 c d x^3 + d^2 x^6}\right] \right)}{3 d \sqrt{-1 + c^2 + 2 c d x^3 + d^2 x^6}}$$

### Problem 40: Result more than twice size of optimal antiderivative.

$$\int x^3 (a + b \operatorname{ArcSec}[c + d x^4]) dx$$

Optimal (type 3, 58 leaves, 7 steps):

$$\frac{a x^4}{4} + \frac{b (c + d x^4) \operatorname{ArcSec}[c + d x^4]}{4 d} - \frac{b \operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{(c + d x^4)^2}}\right]}{4 d}$$

Result (type 3, 137 leaves):

$$\frac{a x^4}{4} + \frac{b (c + d x^4) \operatorname{ArcSec}[c + d x^4]}{4 d} - \frac{b \sqrt{-1 + (c + d x^4)^2} \left( -\operatorname{Log}\left[1 - \frac{c + d x^4}{\sqrt{-1 + (c + d x^4)^2}}\right] + \operatorname{Log}\left[1 + \frac{c + d x^4}{\sqrt{-1 + (c + d x^4)^2}}\right] \right)}{8 d (c + d x^4) \sqrt{1 - \frac{1}{(c + d x^4)^2}}}$$

**Problem 41: Result more than twice size of optimal antiderivative.**

$$\int x^{-1+n} \operatorname{ArcSec}[a + b x^n] dx$$

Optimal (type 3, 49 leaves, 6 steps):

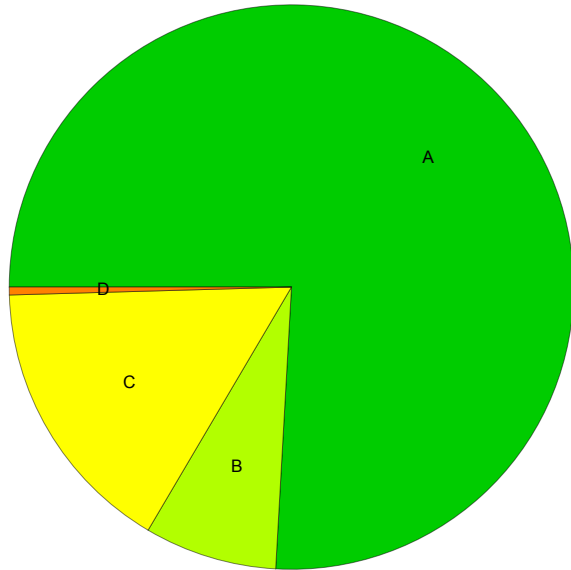
$$\frac{(a + b x^n) \operatorname{ArcSec}[a + b x^n]}{b n} - \frac{\operatorname{ArcTanh}\left[\sqrt{1 - \frac{1}{(a + b x^n)^2}}\right]}{b n}$$

Result (type 3, 130 leaves):

$$\frac{(a + b x^n) \operatorname{ArcSec}[a + b x^n]}{b n} - \frac{\sqrt{-1 + (a + b x^n)^2} \left( -\operatorname{Log}\left[1 - \frac{a + b x^n}{\sqrt{-1 + (a + b x^n)^2}}\right] + \operatorname{Log}\left[1 + \frac{a + b x^n}{\sqrt{-1 + (a + b x^n)^2}}\right] \right)}{2 b n (a + b x^n) \sqrt{1 - \frac{1}{(a + b x^n)^2}}}$$

## Summary of Integration Test Results

224 integration problems



A - 170 optimal antiderivatives

B - 17 more than twice size of optimal antiderivatives

C - 36 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts